

1.2.1 Answers to Exercise.

1. $AB = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{bmatrix}.$

$BA = 30.$

2. $(AB)C = \begin{bmatrix} 3 & 45 & 9 \\ 1 & -11 & 17 \\ 7 & 17 & 13 \end{bmatrix} = A(BC)$

3. $a = -6, b = 8, c = -3, d = 62.$

4. (a) No.

(b) $AB = \begin{bmatrix} 9 & 5 & -6 & -13 \\ -7 & 5 & -2 & -16 \\ -5 & -17 & 14 & 49 \end{bmatrix}, AC = \begin{bmatrix} 9 & 5 & -6 & -13 \\ -7 & 5 & -2 & -16 \\ -5 & -17 & 14 & 49 \end{bmatrix}.$

(c) Yes.

5. (a) $AB = \mathcal{O}$ and $BA = \mathcal{O}.$

(b) —

(c) —

6. —

7. $\begin{bmatrix} 1 \\ 1 \end{bmatrix} [w - y \quad x - z \quad y - w \quad z - x].$

8. A possible choice for a, b, c, d is ' $a = 2, b = -2, c = 3, d = -3$ '.

9. —

10. (a) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c$

(b) i. $[x \quad y \quad 1] \begin{bmatrix} 1 & 9/2 & 5/2 \\ 9/2 & 1 & 4 \\ 5/2 & 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$

iii. $[x \quad y \quad 1] \begin{bmatrix} 0 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 0 & -25 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$

ii. $[x \quad y \quad 1] \begin{bmatrix} 1/9 & 0 & 0 \\ 0 & 1/16 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$

iv. $[x \quad y \quad 1] \begin{bmatrix} 0 & 0 & 6 \\ 0 & 1 & 0 \\ 6 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$

11. $A^2 = \begin{bmatrix} 0 & 0 & a^2 & 2a^3 \\ 0 & 0 & 0 & a^2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$

$A^3 = \begin{bmatrix} 0 & 0 & 0 & a^3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$

$A^4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \mathcal{O}_{4 \times 4}.$

12. (a) $AB = \begin{bmatrix} 6 & 6 & 6 \\ 6 & 6 & 6 \end{bmatrix}.$

(b) $AB = \frac{n(n+1)}{2}A.$

13. (a) False.

(b) True.

14. —

15. (a) —

(b) True.

16. —

17. —

18. (a) Yes.

(b) No. A counter-example is given by $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$

19. —

20. —

21. —