1.1.1 Answers to Exercise.

1. (a) $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$.

   (b) $B = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 6 & 7 & 8 & 9 & 10 \\ 6 & 7 & 8 & 9 & 10 & 11 \\ 7 & 8 & 9 & 10 & 11 & 12 \end{bmatrix}$.

   (c) $C = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$.

   (d) $D = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \end{bmatrix}$.

   (e) $E = \begin{bmatrix} 0 & -1 & -1 & -1 & -1 & -1 \\ 1 & 0 & -1 & -1 & -1 & -1 \\ 1 & 1 & 0 & -1 & -1 & -1 \\ 1 & 1 & 1 & 0 & -1 & -1 \\ 1 & 1 & 1 & 1 & 0 & -1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$.

   (f) $2C = A + D$.

2. (a) $B = \begin{bmatrix} a_{15} & a_{14} & a_{13} & a_{12} & a_{11} \\ a_{25} & a_{24} & a_{23} & a_{22} & a_{21} \\ a_{35} & a_{34} & a_{33} & a_{32} & a_{31} \\ a_{45} & a_{44} & a_{43} & a_{42} & a_{41} \\ a_{55} & a_{54} & a_{53} & a_{52} & a_{51} \end{bmatrix}$

   (b) $C = \begin{bmatrix} a_{55} & a_{54} & a_{53} & a_{52} & a_{51} \\ a_{45} & a_{44} & a_{43} & a_{42} & a_{41} \\ a_{35} & a_{34} & a_{33} & a_{32} & a_{31} \\ a_{25} & a_{24} & a_{23} & a_{22} & a_{21} \\ a_{15} & a_{14} & a_{13} & a_{12} & a_{11} \end{bmatrix}$

3. $r = 1$ only.

4. ——

5. Start in this way:—

   Suppose A is a matrix.

   Denote by $P(n)$ the proposition 'the equality $nA = \underbrace{A + A + A + \cdots + A}_{n \text{ copies of } A}$ holds'.

6. Suppose A, B are $(m \times n)$-matrices.

   Determine all possible $(m \times n)$-matrices X for which $3(X + \frac{1}{2}A) = 5(X - \frac{3}{4}B)$ holds. Leave your answer(s) in terms of A, B.

   $X = \frac{3}{4}A + \frac{15}{8}B$.

7. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

   Determine all possible (3×3)-matrices X for which the equality $X + A = 2(X - B)$ holds.

   $X = A + 2B = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{bmatrix}$