$$
\begin{tikzcd}
 & \text{Systems of linear equations} \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow, "\substack{\text{scalar multiplication} \\ \text{\& addition}}"] & \\
\text{Matrix multiplication} \arrow[rr, leftrightarrow, "\substack{\text{matrix-vector} \\ \text{product}}"'] & & \text{linear combinations}
\end{tikzcd}
$$

Take the following as an example:

$$
(S) := \begin{cases} x + y + z = -1 \\ 2x - 3y + z = 0 \\ 2y - z = 2 \end{cases}
$$

$$
\iff \begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 1 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}
$$

From the above, we can the relation of rules for matrix multiplication & system of linear equations.

On the other hand,

$$S := \begin{cases} x + y + z = 1 \\ 2x - 3y + z = 0 \\ 2y - z = 2 \end{cases}$$

$\Leftrightarrow$ amounts to saying that

$$\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = x \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + y \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} + z \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

i.e $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ "is a linear combination" of

$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ with respect to

scalars, x, y, z.

Hence: To solve (S) amounts to saying to.

we want to know the scalars x, y, z in the

linear combinations

As you can see, all these 3 objects are closely related with each other.

Sometimes it might be easier to transfer from one point view to another to solve your problems.

☆: Calculation is important!

(Try to solve the following problem:

why scalar multiplication can be treated as matrix multiplication? To be more precise, Find a square matrix Λ such that for all square matrix A of size $(n \times n)$

$\lambda A = \Lambda \cdot A$)

There is also one thing I didn't mention on

Thursday's Lecture: commutativity

Now try to solve the following problem:

Given a square matrix (of size $n \times n$)

$$\Lambda = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix}, \qquad \lambda_i \neq \lambda_j \text{ if } i \neq j.$$

find all square matrices of size $n \times n$

which commute with Λ. !!