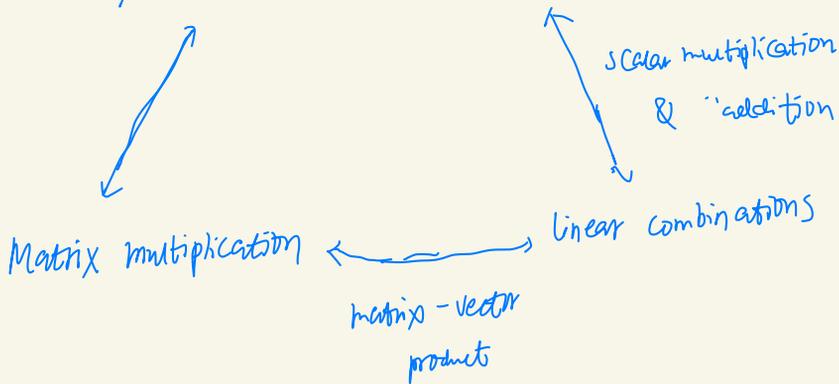


# Systems of linear equations



Take the following as an example:

$$(S): \begin{cases} x + y + z = -1 \\ 2x - 3y + z = 0 \\ xy - z = 2 \end{cases}$$

$$\Leftrightarrow \begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 1 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

from the above, we can see the relation of rules for matrix multiplication & system of linear equations.

On the other hand,

$$S := \begin{cases} x + y + z = 1 \\ 2x - 3y + z = 0 \\ xy - z = 2 \end{cases}$$

↪ amounts to saying that

$$\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = x \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + y \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} + z \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

i.e.  $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$  is a linear combination of

$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$ ,  $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$  with respect to

scalars,  $x, y, z$ .

Hence: To solve (S) amounts to saying to.

We want to know the scalars  $x, y, z$  in the linear combinations.

As you can see; all these 3 objects are closely related with each other.

Sometimes it might be easier to transfer from one point view to another to solve your problems.

☆: Calculation is important!

( Try to solve the following problem:

Why scalar multiplication can be treated as matrix multiplication? To be more precise,

Find a square matrix  $\lambda$  such that for all square matrix  $A$  of size  $(n \times n)$

$$\lambda A = \lambda \cdot A )$$

There is also one thing I didn't mention on

Thursday's Lecture: Commutativity

Now try to solve the following problem:

Given a square matrix (of size  $n \times n$ )

$$A = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix}, \quad \lambda_i \neq \lambda_j \text{ if } i \neq j.$$

find all square matrices of size  $n \times n$

which commute with  $A$ . !!