MATH5010 Linear Analysis: Homework 8. Deadline: 04 Dec 2022

Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard. Please refer to the course web for details.

1. Let M be a vector subspace of a Hilbert space X. Let $M^\perp$ the orthogonal subspace of M. Show that
   (a) $M^\perp$ is closed.
   (b) $(\overline{M})^\perp = M^\perp$.
2. Let H_1 and H_2 be the Hilbert spaces. Let $(e_n)_{n=1}^\infty$ and $(f_n)_{n=1}^\infty$ be the orthonormal bases for H_1 and H_2 respectively. Let $U : H_1 \longrightarrow H_2$ be a linear operator such that $U(e_n) = f_n$ for all $n = 1, 2, ...$ Show that $(Ux, Uy) = (x, y)$ for all $x, y \in H_1$.

$* * *$ **End** $* * *$