THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH6042 - Topics in Differential Equations II - 2022/23 Term 2

Meeting details:

LSB 222 at 2:30pm-5:15pm on Thursday in the following 15 dates —
January: 12, 19
February: 2, 9, 16, 23
March: 2, 9, 16, 23, 30
April: 6, 13, 20, 27

Course description:

The Boltzmann equation is the basis for the kinetic theory of gases and provides a useful tool for the mathematical study of nonequilibrium phenomena in statistical mechanics. A fundamental aspect in kinetic theory is to understand the limiting procedure related to either the validity of the Boltzmann equation from the large number Newtonian particle system or the approximation of the Boltzmann equation through the fluid dynamical system. Meanwhile, the Boltzmann equation itself is of the very importance for describing the motion of a rarefied gas flow in a domain with boundaries. The existence and stability issues on various fundamental physical problems, such as the Couette flow, Poiseuille flow and thermal transpiration, cf. [8], are far from being understood in a rigorous sense in kinetic theory. This topic course provides an opportunity to explore the mathematical foundation of kinetic theory as well as most recent advances of the subject.

Tentatively selected topics (to be updated)

- Introduction ([2, Chapter 8], [11], [12]).
- Spatially homogeneous equation: Existence and regularity ([3, Chapter 4] or [1]).
- Spectral theory ([9]).
- Nonlinear energy method ([6]).
- Global solutions around vacuum ([5, Chapter 2]).
- DiPerna-Lions renormalized solution for general data ([3, Chapter 5.3]).
- Application of Fourier transform to kinetic theory ([4]).
- Limit from kinetic equation to fluid equation ([10],[7]).
- Limit from particle system to kinetic equation ([3, Chapter 4]).
- More topics will be covered if time allows.

Assessment type: Each student should submit a course report on a topic assigned by the course instructor. The assigned topic can be either the content learned in class or a research paper related to the course topics. The details will be announced in due course.

References

[1] Alberto Bressan, Notes on the Boltzmann Equation, http://personal.psu.edu/axb62/PSPDF/boltz.pdf.

[2] C. Cercignani, *The Boltzmann Equation and Its Application.* Springer, Berlin, 1988.

[3] C. Cercignani, R. Illner, M. Pulvirenti, *The Mathematical Theory of Dilute Gases*, Springer-Verlag, New York, 1994.

[4] Laurent Desvillettes, About the use of Fourier transform for the Boltzmann equation, http://laurent.desvillettes.free.fr/por47.pdf.

[5] R.T. Glassey, *The Cauchy Problem in Kinetic Theory*, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, 1996.

[6] Yan Guo, The Boltzmann equation in the whole space. *Indiana Univ. Math. J.* **53** (2004), 1081–1094. https://www.jstor.org/stable/24903533

[7] L. Saint-Raymond, *Hydrodynamic Limits of the Boltzmann Equation.* Lecture Notes in Mathematics, 1971. Springer-Verlag, Berlin, 2009. xii+188

[8] Y. Sone, *Molecular Gas Dynamics: Theory, Techniques, and Applications.* Birkhauser, Boston, 2007.

[9] Seiji Ukai, Solutions of the Boltzmann equation, *Pattern and Waves – Qualitative Analysis of Nonlinear Differential Equations* (eds. M. Mimura and T. Nishida), Studies of Mathematics and Its Applications **18**, pp37-96, Kinokuniya-North-Holland, Tokyo, 1986.

[10] Seiji Ukai, Asymptotic Analysis of Fluid Equations. This is a book chapter in https://link.springer.com/chapter/10.1007%2F11545989_5

[11] C. Villani, A review of mathematical topics in collisional kinetic theory. Handbook of mathematical fluid dynamics, Vol. I, 71–305. North-Holland, Amsterdam, 2002.

[12] C. Villani and C. Mouhot, Kinetic theory, in *The Princeton Companion to Applied Mathematics* (2015).