

THE CHINESE UNIVERSITY OF HONG KONG  
Department of Mathematics  
MATH4240 - Stochastic Processes - 2022/23 Term 2

**Homework 6**

**Due: Tuesday 11 April 2023**

Please submit online via Blackboard your answers to **all TEN questions below** including two supplementary questions. The late submission will not be accepted. Reference solutions will be provided after grading.

**Exercises** (Chapter 3, Page 107): 1, 2, 3, 4, 5, 6, 7, 10

**Supplementary:**

- Q1.** There are three states: 1 =sunny, 2 =smoggy, 3 =rainy. The weather stays sunny for an exponentially distributed number of days with mean 3, then becomes smoggy. It stays smoggy for an exponentially distributed number of days with mean 4, then rain comes. The rain lasts for an exponentially distributed number of days with mean 1, then sunshine returns. Let  $\{X_t\}_{t \geq 0}$  be a Markov jump process to describe the weather. Find the rate matrix  $D$  and Markov matrix  $Q$ .
- Q2.** A factory has three machines in use and one repairman. Suppose each machine works for an exponential amount of time with mean 60 days between breakdowns, but each breakdown requires an exponential repair time with mean 4 days. Let  $\{X_t\}_{t \geq 0}$  be a Markov jump process to describe the number of working machines. Find the rate matrix  $D$  and Markov matrix  $Q$ .