Exercise 1

1. Prove the following result:
(a) let A be an $m \times n$ matrix, $b \in \mathbb{R}^m$, and let $S := \{x \in \mathbb{R}^n : Ax = b\}$. (The set S is just the set of all solutions of the linear equation $Ax = b$.) Then the set S is a convex subset of $\mathbb{R}^n$.
(b) In $\mathbb{R}^n$ the set $H := \{x \in \mathbb{R}^n : a_1x_1 + \ldots + a_nx_n = c\}$ is a convex set. For any particular choice of constants a_i it is a hyperplane in $\mathbb{R}^n$.

2. Prove that if $C \subset \mathbb{R}^n$ is convex, then $c\ell(C)$, the closure of C, is also convex.

3. Prove thatThe intersection of any number of convex sets is convex.