

**THE CHINESE UNIVERSITY OF HONG KONG**

**Department of Mathematics**

**MATH4010 Functional Analysis 2022-23 Term 1**

Homework 4

Deadline: 2022-10-10 Monday

Notice:

- All the assignments must be submitted before the deadline.
- Each assignment should include your name and student ID number.

1. Prove that for every  $x$  in a normed space  $X$ , the following identity holds:

$$\|x\| = \sup \left\{ \frac{|f(x)|}{\|f\|} : f \in X^*, f \neq 0 \right\}.$$

2. Let  $C[0, 1]$  be the vector space of continuous functions on  $[0, 1]$ . Define  $\delta(x) = x(0)$  for  $x \in C[0, 1]$ .

- (a) Show that  $\delta$  is a bounded linear functional if  $C[0, 1]$  is endowed with the sup-norm. Find the norm of  $\delta$ .
- (b) Show that  $\delta$  is an unbounded linear functional if  $C[0, 1]$  is endowed with the norm

$$\|x\| = \int_0^1 |x(t)| dt.$$

— THE END —