MATH3060 HW 7 Due date: Nov 25, 2022 (at 11:00 am)

1. Show that

   (a) for any $x_0 \in [0,1]$ and $y_0 \in \mathbb{R}$,

   $C_{x_0, y_0} = \{ f \in C[0,1] : f(x_0) = y_0 \}$ is nowhere dense in $C[0,1]$;

   (b) $\{ f \in C[0,1] : f(\frac{1}{n}) \in \mathbb{Q}, \forall n = 1,3,3,\cdots \}$ is of 1^{st} category in $C[0,1]$.

2. Using Baire Category Theorem, show that transcendental numbers are dense in $\mathbb{R}$. (Recall: $x \in \mathbb{R}$ is algebraic if x is a root of polynomial with integer coefficients; $x \in \mathbb{R}$ is transcendental if x is not algebraic.)

3. Let X be a metric space, $G_n, n=1,2,\cdots$, are open subsets. Suppose that $G = \bigcap_{n=1}^{\infty} G_n$ is dense in X. Show that G is a residual.

4. Show that a nonempty countable metric space with no isolated point cannot be complete.

(End)