MATH3060 HW6 Due date: Nov 18, 2022 (at 11:00 am)

1. Show that $\{\cos nx\}_{n=1}^{\infty}$ is not equicontinuous in $C[0,1]$.

2. Let $\varphi: \mathbb{R} \to \mathbb{R}$ be uniformly continuous. Show that $\{f_n\}$ defined by $f_n(x) = \varphi(x-n)$ is equicontinuous.

   Is this true if φ is continuous but not uniformly continuous?

3. Let $\{f_n\}$ be a sequence in $C(\mathbb{R})$ such that for any closed and bounded interval I, $\{f_n\}$ is bounded and equicontinuous in $C(I)$. Show that there is a subsequence $\{f_{n_k}\}$ converges pointwisely in $\mathbb{R}$.

   (Hint: Use $\mathbb{R} = \bigcup_{l=1}^{\infty} [-l, l]$ and Cantor's diagonal trick.)

4. Let $\{f_n\}$ be a sequence of Riemann integrable functions on $[0,1]$ such that for some $M > 0$

$$\int_0^1 |f_n|^2 \leq M, \text{ for all } n$$

Using Ascoli's Theorem, show that $\{F_n\}$ is precompact in $(C[0,1], d_\infty)$, where $F_n(x) = \int_0^x f_n(t)\,dt$, $\forall n$.

(End)