Review for Mid-term

Ch6 Differentiation

§6.1 Derivative

Definition, Carathéodory Theorem, Chain rule, and basic properties.

§6.2 Mean Value Theorem

Def. of derivative

$\Rightarrow$ Interior Extremum Theorem

$\Rightarrow$ Rolle's Theorem (special case of Mean Value Thm)

$\Rightarrow$ Mean Value Theorem

Applications of Mean Value Thm:

- Monotonic functions
- First Derivative Test for Extrema
- Approximations
- Inequalities

Also

Interior Extremum Theorem $\Rightarrow$ Darboux's Theorem (Thm 6.2.12)

§6.3 L'Hospital's Rules

(Rolle's Thm ⇒)

Cauchy Mean Value Theorem (generalizing Mean Value Thm)

⇒ L'Hospital's Rules.

§6.4 Taylor's Theorem

(Rolle's Thm ⇒)

Taylor's Theorem with remainder in Lagrange form
(derivative form)

Applications of Taylor's Theorem

- Approximations
- Inequalities
- Higher Derivative Test of Extrema
- Convex function
- Newton's method.

Ch 7 Riemann Integral

§7.1 Riemann integral

- Partitions
- norm of a partition
- Tagged partitions
- Riemann sums.

$\Rightarrow$ Definition of Riemann integrable functions and Riemann integrals.

$\Rightarrow$ Boundedness of integrable functions and basic properties

§6.2 Riemann integrable functions

(Def. of Riemann integrability $\Rightarrow$)
Cauchy Criterion
$\Rightarrow$ Squeeze Theorem
$\Swarrow$ $\Searrow$

Integrability of Continuous functions and monotone functions

Additivity Theorem

Optional Exercises

(1) Suppose f is differentiable on $(0,+\infty)$, and

$$f'(x) \to 0 \quad \text{as} \quad x \to +\infty.$$

Show that $f(x+2) - f(x) \to 0$ as $x \to +\infty$

(2) If f' exists and continuous on $[a,b]$. Then

$\forall \varepsilon > 0$, $\exists \delta > 0$ such that

$$\left| \frac{f(y)-f(x)}{y-x} - f'(x) \right| < \varepsilon$$

$\forall\ 0 < |y-x| < \delta$ and $x, y \in [a,b]$.

(3) For $f: \mathbb{R} \to \mathbb{R}$, a point $c \in \mathbb{R}$ is called a fixed point of f if $f(c) = c$.

Suppose f is differentiable and $f'(x) \neq 1$, $\forall x \in \mathbb{R}$.

Show that f has at most one fixed point.

(4) Suppose that f is bounded on $[a,b]$ and $f^2 \in \mathcal{R}[a,b]$.

Is it true that $f \in \mathcal{R}[a,b]$?

(End)