THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH 2058 Honours Mathematical Analysis I 2022-23
Tutorial 3
29th September 2022

- Tutorial problems will be posted every Wednesday, provided there is a tutorial class on the Thursday same week. You are advised to try out the problems before attending tutorial classes, where the questions will be discussed.
- Solutions to tutorial problems will be posted after tutorial classes.
- If you have any questions, please contact Eddie Lam via echlam@math.cuhk.edu.hk or in person during office hours.

1. Compute and prove the following limits using ϵ arguments.
   (a) $x_n = \frac{3n+1}{2n+5}$.
   (b) $x_n = \frac{n^2-1}{2n^2+3}$.
   (c) $x_n = \sqrt{4n^2+n} - 2n$
   (d) $x_n = na^n$ where $0 < a < 1$.
   (e) $x_n = b^{\frac{1}{n}}$ where $b > 1$.
   (f) $x_n = c^{\frac{1}{n}}$ where $1 > c > 0$.
   (g) $x_n = n^{\frac{1}{n}}$.
2. Define (x_n) recursively by $x_1 = 2$ and $x_{n+1} = \frac{1}{2}(x_n + \frac{2}{x_n})$, prove that $\lim x_n$ exists and is equal to $\sqrt{2}$.
3. Suppose that (x_n) is a positive sequence where $\lim \frac{x_{n+1}}{x_n} = c < 1$, prove that $\lim x_n = 0$.
4. Suppose that there is a sequence (x_n) so that $|x_n - x_{n+1}| \leq \frac{1}{n}$, does (x_n) necessarily converge?
5. Suppose that the sequence (x_n) satisfies the property that there is some $\delta > 0$ and $k, N \in \mathbb{N}$ so that $\delta < x_n < n^k$ for $n \geq N$. Prove that $\lim \sqrt[n]{x_n} = 1$.
6. (Cesàro sum) Suppose that $\lim x_n = L$ exists, prove that $y_n = \frac{1}{n}(x_1 + ... + x_n)$ is also convergent with the same limit. Find a counter-example for the converse statement, i.e. exhibit a divergent sequence (x_n) so that the associated (y_n) is convergent.