MATH 2050C Lecture 19 (Mar 30)

[Problem Set 10 posted, due on Apr 7, 2023.]

[Quiz 4 on Apr 6, 2023.]

Q: How to construct NEW cts fcn from OLD ones?

A: "most of the time" use limit theorems. (§5.2 in textbook)

Thm 1: $f, g : A \to \mathbb{R}$ is cts (at $c \in A$)

$\Rightarrow f \pm g, fg, f/g$ is cts (at $c \in A$) wherever they are defined

$g(x) = x \rightsquigarrow \frac{1}{g}(x) = \frac{1}{x}$

cts everywhere — cts everywhere it is defined, i.e. $x \neq 0$

Thm 2: $f : A \to \mathbb{R}$ is cts (at $c \in A$)

$\Rightarrow \sqrt{f}, |f|$ are cts (at $c \in A$) wherever they are defined.

Thm 3: (Composition of functions)

If f is cts at $c \in A$, and

g is cts at $f(c) \in B$,

then $g \circ f$ is cts at $c \in A$.

$f : A \to \mathbb{R}$

$g : B \to \mathbb{R}$

and $f(A) \subseteq B$

$\Rightarrow g \circ f : A \to \mathbb{R}$

$g \circ f(x) := g(f(x))$

<u>Proof</u>: "Use ε-δ def^{n}": Let $b := f(c) \in B$

Let $\varepsilon > 0$ be fixed but arbitrary.

Since g is cts at $b = f(c)$, then $\exists\ \delta_1 = \delta_1(\varepsilon) > 0$ s.t.

$(\dagger)$ ...... $|g(y) - g(b)| < \varepsilon$ when $y \in B$, $|y - b| < \delta_1$

Since f is cts at $c \in A$, for the $\delta_1 > 0$ (new ε), $\exists\ \delta_2 = \delta_2(\delta_1) > 0$ s.t.

$(\dagger\dagger)$ ...... $|f(x) - f(c)| < \delta_1$ when $x \in A$, $|x - c| < \delta_2$

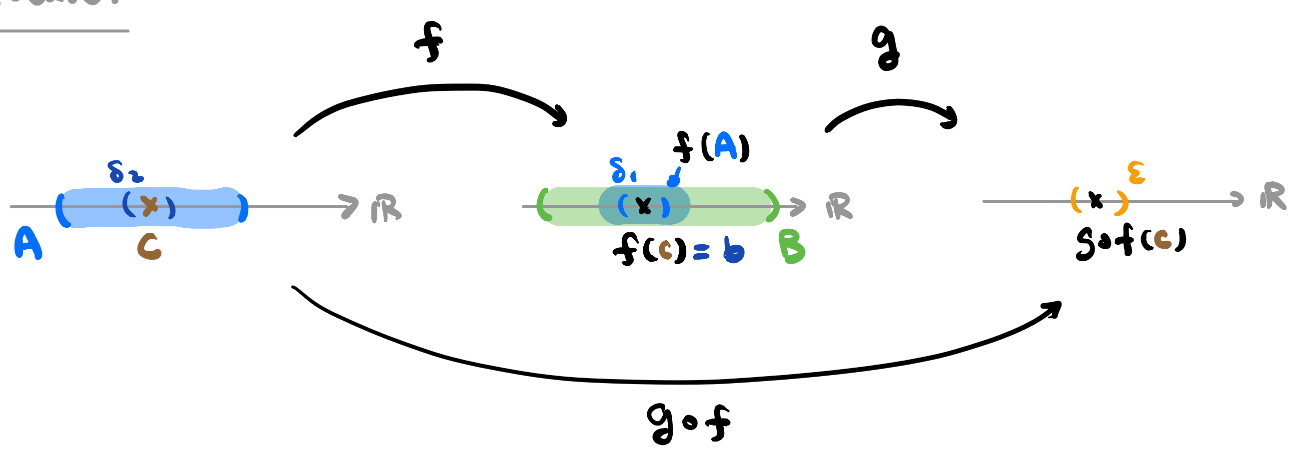
For such $\delta_2 > 0$, when $x \in A$, $|x - c| < \delta_2$

by $(\dagger\dagger)$, $\quad |\underbrace{f(x)}_{y} - \underbrace{f(c)}_{b:=}| < \delta_1$

by $(\dagger)$, $\quad |\underbrace{g(f(x))}_{g\circ f(x)} - \underbrace{g(f(c))}_{g\circ f(c)}| < \varepsilon$

$\square$

<u>Picture</u>:



<u>Exercise</u>: Prove this using sequential criteria.