

MATH 2050C Mathematical Analysis I
2022-23 Term 2
Problem Set 12

due on Apr 21, 2023 (Friday) at 11:59PM

Instructions: You are allowed to discuss with your classmates or seek help from the TAs but you are required to write/type up your own solutions. You can either type up your assignment or scan a copy of your written assignment into ONE PDF file and submit through Gradescope on/before the due date. Please remember to write down your name and student ID. **No late homework will be accepted.** All the exercises below are taken from the textbook.

Required Readings: Chapter 5.4

Optional Readings: Chapter 5.6

Problems to hand in

Section 5.4: Exercise # 2, 6, 8, 12

Suggested Exercises

Section 5.4: Exercise # 1, 3, 4, 5, 7, 9, 10, 11, 14, 15

Challenging Exercises (optional)

1. Section 5.4: Exercise # 13, 16
2. Let $f : \mathbb{Q} \rightarrow \mathbb{R}$ be a uniformly continuous function. Prove that there exists a continuous function $g : \mathbb{R} \rightarrow \mathbb{R}$ such that $g(x) = f(x)$ for all $x \in \mathbb{Q}$.
3. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $|f(x)| \leq 1$ for all $x \in \mathbb{R}$. Suppose $g : [-1, 1] \rightarrow \mathbb{R}$ is a continuous injective function. Prove that f is (uniformly) continuous if $g \circ f$ is (uniformly) continuous.