2022-23 MATH2048: Honours Linear Algebra II
Homework 4

Due: 2022-10-07 (Friday) 23:59

For the following homework questions, please give reasons in your solutions. Scan your solutions and submit it via the Blackboard system before due date.

1. (a) Find two linear transformations $T : V \longrightarrow W$ and $U : W \longrightarrow V$ such that $UT = T_0$ (the zero transformation from V to V) but $TU \neq T_0$ (the zero transformation from W to W).
   (b) Based on T and U in (a), find two matrices A and B such that $AB = O$ but $BA \neq O$.

2. Let $g_0(x) = x + 1$. Let $T : P_2(\mathbb{R}) \longrightarrow P_3(\mathbb{R})$ and $U : P_3(\mathbb{R}) \longrightarrow \mathbb{R}^3$ be defined by
$$T(f(x)) = f'(x)g_0(x) + \int_0^x f(t)dt \text{ and } U(h(x)) = (h(0), h(1), h'(1))$$
   Let α, β, γ be the standard ordered bases for $P_2(\mathbb{R}), P_3(\mathbb{R}), \mathbb{R}^3$ respectively.
   (a) Compute $[T]_\alpha^\beta$, $[U]_\beta^\gamma$, $[U]_\beta^\gamma[T]_\alpha^\beta$ and $[UT]_\alpha^\gamma$.
   (b) Let $h_0(x) = 1 - 2x - x^2 + x^3$, compute $[h_0(x)]_\beta$, $[U]_\beta^\gamma[h_0(x)]_\beta$ and $[U(h_0(x))]_\gamma$.

3. Sec. 2.3: Q17

4. Let V and W be two finite-dimensional vector spaces, and let $T : V \longrightarrow W$ be a linear transformation. Suppose β is a basis for V. Prove that T is invertible if and only if $T(\beta)$ is a basis for W.

5. Sec. 2.4: Q16

The following are extra recommended exercises not included in homework.

1. Sec. 2.3: Q11
2. Sec. 2.4: Q9
3. Sec. 2.4: Q17