

## Assignment 9

Coverage: 16.4 in Text.

Exercises: 16.3 no 32, 33, 38; 16.4 no 7, 11, 14, 23, 26, 28, 35, 37, 39.

Hand in 16.4 no 14, 28, 35, 39 and Supplementary Problem no. 1 by April 4.

### Supplementary Problems

1. Let  $D$  be the parallelogram formed by the lines  $x + y = 1$ ,  $x + y = 3$ ,  $y = 2x - 3$ ,  $y = 2x + 2$ . Evaluate the line integral

$$\oint_C dx + 3xy dy$$

where  $C$  is the boundary of  $D$  oriented in anticlockwise direction. Suggestion: Try Green's theorem and then apply change of variables formula.

2. Find a potential for the vector field

$$\frac{-y}{x^2 + y^2} \mathbf{i} + \frac{x}{x^2 + y^2} \mathbf{j} ,$$

in the region obtained by deleting the line  $(x, 0), x \leq 0$ , from  $\mathbb{R}^2$ .

3. Let  $F = M\mathbf{i} + N\mathbf{j}$  be a smooth vector field which is defined in  $\mathbb{R}^2$  except at the origin. Suppose that it satisfies the component test  $M_y = N_x$ . Show that for any simple closed curve  $\gamma$  enclosing the origin and oriented in positive direction, one has

$$\oint_{\gamma} M dx + N dy = \varepsilon \int_0^{2\pi} [-M(\varepsilon \cos \theta, \varepsilon \sin \theta) \sin \theta + N(\varepsilon \cos \theta, \varepsilon \sin \theta) \cos \theta] d\theta ,$$

for all sufficiently small  $\varepsilon$ . What happens when  $\gamma$  does not enclose the origin?