

Assignment 8

Coverage: 16.2, 16.3 in Text.

Exercises: 16.2 no 29, 32, 37, 41, 42, 43. 16.3 no 2, 5, 9, 11, 15, 16, 18, 20, 27, 29, 32.

Hand in 16.2 no 32, 42; 16.3 no 11, 20, 32 by March 28.

Supplementary Problems

1. Let $F = (F_1, \dots, F_n)$ be a smooth vector field in an open region in \mathbb{R}^n . Show that if it is conservative, then the necessary conditions (Component Test) hold

$$\frac{\partial F_i}{\partial x_j} = \frac{\partial F_j}{\partial x_i}, \quad \forall i, j.$$

2. Let \mathbf{F} be a smooth vector field in the entire space \mathbb{R}^n . Show that

$$\Phi(x, y, z) = \int_0^1 \mathbf{F}(tx, ty, tz) \cdot (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) dt,$$

defines a potential function for \mathbf{F} provided it passes the component test.

3. Let C be the oriented curve runs from the origin to $(2, 0)$ along the cardioid $r = 1 + \cos \theta$ in the upper half plane. Find the work done of $\mathbf{F} = (\sin xy + xy \cos xy)\mathbf{i} + x^2 \cos xy\mathbf{j}$ along C .