Assignment 10

Coverage: 16.5 (skip"Implicit Surfaces") and 16.6.

Exercises: 16.5 no 4, 8, 10, 13, 20, 24, 31, 32, 33, 38, 42, 45; 16.6 no 4, 8, 10, 13, 24, 27, 32, 38.
Hand in 16.5 no 8,31, 33; 16.6 no 8, 13, 38 by April 18.

Supplementary Problems

1. Consider the parametric surface

$$\mathbf{r}(u,v) = (u+6v, -2u-12v+5, -1), \quad (u,v) \in [0,1]\times[0,1]\ .$$

   Is it a smooth surface? Describe its image. Recall that by definition a parametric surface is smooth if $\mathbf{r}$ is continuously differentiable and $\mathbf{r}_u \times \mathbf{r}_v$ is linearly independent in the interior of the region of definition.

2. Let S be the graph $\{(x,y,f(x,y)) :\ (x,y)\in D\}$ where D is a plane region. Show that its surface area is given by

$$\iint_D \sqrt{1+f_x^2+f_y^2}\, dA(x,y)\ .$$

3. Let S be the surface of revolution obtained by rotating $(\varphi(z), z), \varphi(z) > 0, z \in [a,b]$ around the z-axis. Show that its surface area is given by

$$2\pi \int_a^b \varphi(z)\sqrt{1+\varphi'^2(z)}\, dz\ .$$