MATH 6231 : Topic in Optimization Theory.

1. Static Optimization.

2. Dynamic optimal control : deterministic case.

3. Dynamic Optimal Control : stochastic case :

Fleming / Rishel : Deterministic and Stochastic optimal control.

1. static : $\sup\limits_{x \in \mathbb{R}^n} f(x)$. for some function $f : \mathbb{R}^n \to \mathbb{R}$.

$\to \sup\limits_{x \in K} f(x)$ for some $K \subseteq \mathbb{R}^n$.

$K := \{ x \in \mathbb{R}^n \; g_i(x) \leq 0, \quad h_j(x) = 0 \}$

$i = 1, \cdots, m, \qquad j = 1, \cdots, l.$

- Existence. $x^* = \arg\max_{x \in K} f(x)$.
- characterization of x^* / $f(x^*) = \sup_{x \in \partial K} f(x)$.

2: Dynamic. control: / deterministic case.

- Dynamic system: (described by differential equation)

$$X_t = x_0 + \int_0^t b(s, X_s, \alpha_s)\, ds. \quad / \text{ODE}. \quad \dot{X}(t) = b(t, X(t), \alpha(t))$$

$$\Leftrightarrow\!\!\approx \quad X_{t_{i+1}} \approx X_{t_i} + b(t_i, X_{t_i}, \alpha_{t_i})\, \Delta t.$$

$X_t \in \mathbb{R}^d$.

- Objective: $\sup_{(\alpha_t)_{t \geq 0}} \left[\int_0^T L(s, X_s^\alpha, \alpha_s)\, ds + g(X_T^\alpha) \right]$ → subjective to some constraint.

3. Stochastic control: → Stochastic Differential Equation.

$$X_t = x_0 + \int_0^t b(s, X_s, \alpha_s)\, ds + W_t.$$

noise.
Brownian motion.

$$\underbrace{\sup_{\alpha} \mathbb{E}\left[\int_0^T L(s, X_s^{\alpha}, \alpha_s)\, ds + g(X_T^{\alpha}) \right]} \longrightarrow \text{subject to constraints.}$$

Assesement: $\to$ Exam 50%

$\to$ Project $\to$ oral presentation.
$\to$ short report.

(group of 2 persons.)