MATH3060 HW5 Due date: Nov 10, 2021 (at 12:00 noon)

1. Let (X, d), (Y, ρ) be metric spaces. A map $f: E \to Y$ from a subset E of X to Y is called <u>uniform continuous</u> if $\forall \varepsilon > 0$, $\exists \delta > 0$ such that

$$\rho(f(x_1), f(x_2)) < \varepsilon \quad \text{for all } x_1, x_2 \in E \text{ with } d(x_1, x_2) < \delta.$$

Now suppose that (Y, ρ) is complete. Show that for any uniform continuous $f: E \to Y$, there exists a uniform continuous $F: \overline{E} \to Y$ defined on the closure of E in (X, d) such that $F|_E = f$.

2. Show that the equation $x - 3x\sin x + x^4 = 0.001$ has a solution near $x = 0$.

3. Let $\Phi: \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $\Phi\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + y^4 \\ y - x^2 \end{pmatrix}$.

Show that $\Phi\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0.01 \end{pmatrix}$ has a solution.

4. Let $A = (a_{ij})_{n \times n}$ be a matrix. Suppose that

$$\max_{i=1,\dots,n} \sum_{j=1}^{n} |a_{ij}| < 1$$

Then $I - A$ is invertible, where I = identity matrix.

(End)