MATH2058: Analysis I 2021-2022 **Revision Test 2**

Duration: 30 min 28 Oct 2021 8:30 - 9:00

Answer ALL Questions Full Mark: 30

1 (15 marks)**.** Let (x_n) be a sequence in $\mathbb{R}$. We say that (x_n) diverges to $+\infty$ and write $\lim x_n = +\infty$ if for all $M > 0$, there exists $N \in \mathbb{N}$ such that $x_n \geq M$ for all $n \geq N$.

a) Let $x_n := n/\sqrt{n+1}$. Show that $\lim x_n = +\infty$ by definition.

b) Let (x_n) and (y_n) be sequences of postive numbers such that $\lim \dfrac{x_n}{y_n} = +\infty$. Show that if $\lim y_n = +\infty$ then $\lim x_n = +\infty$.

c) Is the converse of part (b) true? Prove your assertion.

2 (15 marks)**.** Let (x_n) be a sequence. We denote $c(x_n) := \frac{1}{n}(x_1 + \cdots + x_n)$ for all $n \in \mathbb{N}$.

a) Find an example of a sequence (x_n) such that $\lim c(x_n)$ exists but $\lim x_n$ does not.

b) Show that in general if $\lim x_n$ exists then $\lim c(x_n)$ exists.