MATH2048: Honours Linear Algebra II

Homework 1

Due: 2021-09-17 (Friday) 23:59

10 pts for each of the 10 questions. Please give reasons in your solutions.

1. Textbook (Friedberg). Sec. 1.2: Q12
2. Textbook (Friedberg). Sec. 1.3: Q19
3. Textbook (Friedberg). Sec. 1.3: Q28
4. Textbook (Friedberg). Sec. 1.4: Q15
5. Textbook (Friedberg). Sec. 1.5: Q15
6. Textbook (Friedberg). Sec. 1.6: Q24
7. Textbook (Friedberg). Sec. 1.6: Q29(a)
8. Textbook (Friedberg). Sec. 1.6: Q33b
9. (Quotient Space)

   Let V be a vector space over F and W be a subspace of V. Prove that for any $\vec{v}_1, \vec{v}_2 \in V$, suppose $\vec{v}_1 + W = \vec{v}_2 + W$, then for any $\vec{v}_3 \in V$ satisfying $\vec{v}_3 + W \in V/M$, the following are true:

   (a) $(\vec{v}_1 + W) + (\vec{v}_3 + W) = (\vec{v}_2 + W) + (\vec{v}_3 + W)$

   (b) $a \cdot (\vec{v}_1 + W) = a \cdot (\vec{v}_2 + W)$

10. (Quotient Space)

    Let V be a vector space over F and W be a subspace of V. Let the quotient space of V module W be $V/M = \{\vec{v} + M : \vec{v} \in V\}$. Prove that if the addition and scalar multiplication operations are defined as

    - $(\vec{v}_1 + W) + (\vec{v}_2 + W) := (\vec{v}_1 + \vec{v}_2) + W$
    - $a \cdot (\vec{v} + W) := a \cdot \vec{v} + W$

    V/M is a vector space over F .