Assignment 8

Coverage: 16.1, 16.2 (most) in Text.

Exercises: 16.1 no 12, 13, 15, 21, 25, 27, 29, 34. 16.2 no 11, 16, 20, 24, 25.

Hand in 16.1 no 15, 25, 16.2 no 20, 24 by March 15.

Supplementary Problems

1. Let $\mathbf{r}_1$ and $\mathbf{r}_2$ be on $[a,b]$ and $[\alpha,\beta]$ respectively that describe the same curve C. It has been shown that there exists some φ maps $[a,b]$ one-to-one onto $[\alpha,\beta]$, $\varphi'(t) > 0$, such that $\mathbf{r}_2(\varphi(t)) = \mathbf{r}_1(t)$ when both parametrization runs in the same direction. When they runs in different direction, $\varphi'(t) < 0$. Using this fact to prove that when $\varphi'(t) > 0$,

$$\int_a^b \mathbf{F}(\mathbf{r}_1(t)) \cdot \mathbf{r}_1'(t)\, dt = \int_\alpha^\beta \mathbf{F}(\mathbf{r}_2(\tau)) \cdot \mathbf{r}_2'(\tau)\, d\tau \ .$$

When $\varphi'(t) < 0$,

$$\int_a^b \mathbf{F}(\mathbf{r}_1(t)) \cdot \mathbf{r}_1'(t)\, dt = -\int_\alpha^\beta \mathbf{F}(\mathbf{r}_2(\tau)) \cdot \mathbf{r}_2'(\tau)\, d\tau \ .$$