

MATH1520AB 2021-22 Quiz 4 (week 7)

Full marks: 10 marks

Time allowed: 15 minutes

1. Let $y = f(x)$ be a differentiable function of x that satisfies the equation $x^3 - 2x^2y + y^2 = 0$.

Find $\frac{dy}{dx}$ as a function of x and y .

Answer.

$$\begin{aligned}x^3 - 2x^2y + y^2 &= 0 \\3x^2 - 4xy - 2x^2 \frac{dy}{dx} + 2y \frac{dy}{dx} &= 0 \\(2x^2 - 2y) \frac{dy}{dx} &= 3x^2 - 4xy \\ \frac{dy}{dx} &= \frac{3x^2 - 4xy}{2x^2 - 2y} = \frac{x(3x - 4y)}{2(x^2 - y)}\end{aligned}$$

2. Evaluate the following limits.

(a) $\lim_{x \rightarrow 1} \frac{e^x - e}{1 - x}$

(b) $\lim_{x \rightarrow \infty} \frac{\ln(1 + x^3)}{x^2}$

Answer.

(a) $\lim_{x \rightarrow 1} \frac{e^x - e}{1 - x} = \lim_{x \rightarrow 1} \frac{e^x}{-1} = -e$

(b) $\lim_{x \rightarrow \infty} \frac{\ln(1 + x^3)}{x^2} = \lim_{x \rightarrow \infty} \frac{\frac{3x^2}{1+x^3}}{2x} = \lim_{x \rightarrow \infty} \frac{3x}{2(1+x^3)} = \lim_{x \rightarrow \infty} \frac{\frac{3}{x^2}}{2(\frac{1}{x^3} + 1)} = 0$

3. Let $f(x) = 4x^3 + 6x^2 - 72x + 5$. Find all its relative maxima and relative minima.

Answer.

Since $f'(x) = 12x^2 + 12x - 72 = 12(x + 3)(x - 2)$, the critical numbers are solutions of $f'(x) = 0$, i.e. $x = -3$ or $x = 2$.

x	$(-\infty, -3)$	-3	$(-3, 2)$	2	$(2, \infty)$
$f'(x)$	$+$	0	$-$	0	$+$

Relative maximum: $(-3, 167)$

Relative minimum: $(2, -83)$