MATH1050 Proof-writing Exercise 6 Index of Comments

1. **AIT.**
   You have confused the conjunction '*and*', the conditional '*if ... then ...*'.
   '$H \wedge K$', '$H \longrightarrow K$' are distinct statements.
   Compare this pair of statements:

   (a) '*If John visits Mary then John brings Mary to the cinema.*'
   (b) '*John visits Mary and John brings Mary to the cinema.*'.

   They are saying different things.
   The definition for '$S \subset T$' reads:

   - '*For any object x, if $x \in S$ then $x \in T$.*'

   It is not '*For any object x, $x \in S$ and $x \in T$.*'
   It is logically wrong to try to argue for $A \cup B \subset C \cup D$ in this way:

   - *Pick any x. So-and-so-and-so-and-so. Then $x \in A \cup B$.*
     So-and-so-and-so-and-so. Then $x \in C \cup D$.
     Therefore $A \cup B \subset C \cup D$.

   It is logically wrong to write a sentence such as this one, or anything to such an effect, (even if everything prior to this sentence is correct and relevant in an argument for '$A \cup B \subset C \cup D$'):

   - '*Because ($x \in A \cup B$ and $x \in C \cup D$), we have $A \cup B \subset C \cup D$.*'

2. **AVO.**
   You have confused the conjunction '*and*', the disjunction '*or*'.
   Consider this passage:

   - *... We have $x \in A$ or $x \in B$. In particular $x \in A$. ...*

   The use of this '*in particular*' is logically wrong. What this passage suggests is this wrong reasoning: '*Since ($x \in A$ or $x \in B$), we have $x \in A$.*

   A statement with the disjunction '*or*' such as '$x \in A$ *or* $x \in B$' gives rise to various cases '$x \in A$', '$x \in B$'. To proceed with an argument within the case '$x \in A$', you should continue with, for instance '*(Case 1.) Suppose $x \in A$*'.

3. **CASE.**
   You should indicate clearly to the reader that you are 'splitting' the argument into various cases, in each of which there may be specific extra assumptions.

4. **ESS.**
   In arguing for a subset relation, such as '$S \subset T$' (under some assumptions on S, T), you should (simply) start with these words (after the assumptions on S, T have been stated):

   - *Pick any object x. Suppose $x \in S$.* [Now argue for '$x \in T$'.]

   There is no need (to waste time and ink) on the scenario in which $S = \emptyset$.
   Recall that by definition, '$S \subset T$' reads:

   - '*For any object x, if $x \in S$ then $x \in T$.*'

   In the scenario in which $S = \emptyset$, the statement '$x \in S$' is false. Then the statement '*For any object x, if $x \in S$ then $x \in T$*' is trivially true.

5. **FOR.**
   There are too many meanings for the word '*for*'. Choose an appropriate word, other than '*for*' to indicate what you actually means. (You do not want the reader to read your passage in such a way that you don't intend.)

6. **MA.**
   At least part of the assumptions is missing. But you are going to use these assumptions in the argument. The reader is not responsible to write out the missing assumptions for you, and will simply regard your argument as wrong when you are applying the 'missing assumptions'.

7. **MC.**
   There is a missing case. Your subsequent argument is therefore incomplete and/or wrong.

8. **MS.**
   There is a missing step which you should not have skipped.

9. **NAAA.**
   You have not argued at all for what you claim you are deducing.

   Consider such 'arguments' for '$A \cup B \subset C \cup D$' (under the assumption '$A \subset C$ *and* $B \subset D$') below:

   (a) *... Suppose* $A \subset C$ *and* $B \subset D$.
   Pick any $x \in A$. *... So-and-so-and-so-and-so ... Then we have* $x \in C$.
   Pick any $y \in B$. *... So-and-so-and-so-and-so ... Then we have* $y \in D$.
   Let $u = x$ *or* $u = y$. *... So-and-so-and-so-and-so ... It follows that* $A \cup B \subset C \cup D$.

   (b) *... Suppose* $A \subset C$ *and* $B \subset D$.
   Pick any $x \in A$. *... So-and-so-and-so-and-so ... Then we have* $x \in C$.
   Pick any $y \in B$. *... So-and-so-and-so-and-so ... Then we have* $y \in D$.
   Let $x \in A$ *or* $y \in B$. *... So-and-so-and-so-and-so ...* $A \cup B \subset C \cup D$.

   (c) *... Suppose* $A \subset C$ *and* $B \subset D$.
   Pick any $x \in A$. *... So-and-so-and-so-and-so ... Then we have* $x \in C$.
   Pick any $y \in B$. *... So-and-so-and-so-and-so ... Then we have* $y \in D$.
   Let $x \in A$ *and* $y \in B$. *... So-and-so-and-so-and-so ...* $A \cup B \subset C \cup D$.

   Such 'arguments' for '$A \cup B \subset C \cup D$' are wrong. The problem is that you have probably forgotten where to focus on in the chain of symbols '$A \cup B \subset C \cup D$'. It is, and it should be, the symbol '$\subset$'. As a consequence, you are working on wrong premises. So you have no chance in arriving logically correctly at the desired conclusion.

   The statement that should be argued for (under the assumption '$A \subset C$ *and* $B \subset D$') reads:

   - '*For any object* x, *if* $x \in A \cup B$ *then* $x \in C \cup D$.'

   So the correct opening to the argument, after stating the assumption '*Suppose* $A \subset C$ *and* $B \subset D$.' is:

   - '*Pick any* x. *Suppose* $x \in A \cup B$.' [Now proceed to deduce $x \in C \cup D$.]

   Refer to the notes to find out what you are expected to do to give a correct argument.

10. **PC.**
    Punctuation and capitals should be used appropriately so as to indicate to the reader how the passage is to be read. Omissions may result in the reader being confused with the logic and/or the mathematical content in what you are writing.

11. **RO.**
    Re-organize your argument.

    - If you intend to argue for something like the subset relation '$A \cup B \subset C \cup D$' (under the assumption '$A \subset C$ and $B \subset D$'), you should make focus on this subset relation in your argument, (immediately after stating the assumption '*Suppose* $A \subset C$ *and* $B \subset D$.') Make it clear you work according to the definition of subset. The corresponding opening should be:

      '*Pick any object* x. *Suppose* $x \in A \cup B$.'

      After these words it should be an argument for '$x \in C \cup D$', using '$A \subset C$', '$B \subset D$', and '$x \in A \cup B$'.

    Refer to the notes on the corresponding lecture for reference.

12. **SUP.**
    Use the word 'suppose' for indicating what you are supposing (or supposing in extra) here.

13. **UP.**
    It is useless and pointless to write these lines here. This portion should be appropriately incorporated in the argument for what you intend to deduce.

    For example, if it is '$A \cup B \subset C \cup D$' that you want to deduce (under some other assumptions), it is no good to ask the reader to find out why those 'useless and pointless lines' (like '*for any* x, *if* $x \in A$ *then* $x \in C$') placed above help the argument. The reader is not responsible for doing the task of incorporation for you.

14. **WD.**
    The deduction at this place is wrong.

    Consider the passage below in an 'argument' for '$A \cup B \subset C \cup D$' under the assumption '$A \subset C$ *and* $C \subset D$':

... Pick any x. Suppose $x \in A \cup B$. Then $x \in A$ or $x \in B$.

(Case 1). Suppose $x \in A$. Then ... . Therefore $x \in C \cup D$. It follows that $A \cup B \subset C \cup D$.

...

All that has been achieved at the end of '(Case 1)' (in which $x \in A$ is supposed) is only '*for any x, if ($x \in A \cup B$ and) $x \in A$ then $x \in C \cup D$.*'

So it is logically wrong to 'draw conclusion *It follows that $A \cup B \subset C \cup D$*'. This is because '$A \cup B \subset C \cup D$' reads: '*for any x, if $x \in A \cup B$ then $x \in C \cup D$.*' You have not (and cannot have) justified the latter within the specific case in which the extra assumption '$x \in A$' has been imposed.