MATH1050 Proof-writing Exercise 6 (Answers and selected solution)

1. (a) **Answer.**

   i. Let A, B be sets. The set A is said to be a subset of the set B if the statement (†) holds:
   (†): For any object x, if $x \in A$ then $x \in B$.

   ii. Let A, B be sets. The set A is not a subset of the set B iff the statement (∼†) holds:
   (∼†): There exists some object x_0 such that $x_0 \in A$ and $x_0 \notin B$.

   (b) **Solution.**
   Let $A = \{\zeta \in \mathbb{C} : |\zeta - i| < 1\}, B = \{\zeta \in \mathbb{C} : |\zeta + i| < 3\}$.

   i. Pick any $\zeta \in A$.
   We have $|\zeta - i| < 1$ (by the definition of A).
   By the Triangle Inequality, we have $|\zeta + i| = |\zeta - i + 2i| \leq |\zeta - i| + |2i| = |\zeta - i| + 2 < 1 + 2 = 3$.
   Then $|\zeta + i| < 3$. Therefore, we have $\zeta \in B$ (by the definition of B).
   It follows that $A \subset B$.

   ii. Take $\zeta_0 = 0$.
   We have $\zeta_0 \in \mathbb{C}$.
   Note that $|\zeta_0 + i| = |0 + i| = 1 < 3$.
   Then $\zeta_0 \in B$.
   We verify that $\zeta_0 \notin A$:
   - We have $|\zeta_0 - i| = |0 - i| = 1 \geq 1$.
     Then $\zeta_0 \notin A$.

   It follows that $B \not\subset A$.

2. (a) **Answer.**
   Let A, B be sets.
   The union of the sets A, B is defined to be the set $\{x \mid x \in A \text{ or } x \in B\}$.

   (b) **Solution.**
   Let A, B, C, D be sets. Suppose $A \subset C$ and $B \subset D$.
   Pick any object x.
   Suppose $x \in A \cup B$. Then $x \in A$ or $x \in B$.
   - (Case 1). Suppose $x \in A$. Then, since $A \subset C$, we have $x \in C$. Therefore $x \in C$ or $x \in D$. Hence $x \in C \cup D$.
   - (Case 2). Suppose $x \notin A$. Then $x \in B$. Therefore, since $B \subset D$, we have $x \in D$. Then $x \in C$ or $x \in D$. Hence $x \in C \cup D$.

   Hence in any case, we have $x \in C \cup D$.
   It follows that $A \cup B \subset C \cup D$.

3. ——

4. ——