MATH1050 Proof-writing Exercise 6

Advice.

- Study the Handouts *Examples of proofs concerned with 'subset relations'*, *Examples of proofs for properties of basic set operations* before answering the questions.
- Besides the handouts mentioned above, Questions (9), (12a) of Exercise 6 may also be relevant to some of the questions below.
- When giving an argument, remember to adhere to definition, always. And don't forget to make the argument self-contained: the assumptions needed in the argument should be stated, (likely) at the the top.

1. (a) i. Explain the phrase *subset of a set* by giving its appropriate definition.
   ii. Explain the phrase *being not a subset of a set*, in terms of the notion of *belonging*, by giving an appropriate mathematical statement (whose content is self-contained).

   (b) Let $A = \{\zeta \in \mathbb{C} : |\zeta - i| < 1\}, B = \{\zeta \in \mathbb{C} : |\zeta + i| < 3\}$.
   i. Is it true that $A \subset B$? Justify your answer.
   ii. Is it true that $B \subset A$? Justify your answer.

2. (a) Explain the phrase *union of two sets* by stating its appropriate definition.

   (b) Prove statement below, with reference to the definitions of set equality, subset relation, intersection, union, complement, where appropriate.
   - *Let A, B, C, D be sets. Suppose $A \subset C$ and $B \subset D$. Then $A \cup B \subset C \cup D$.*

3. (a) Explain the phrases *intersection of two sets*, *complement of a set in another (not distinct) set* by stating their appropriate definitions.

   (b) Prove the statements below, with reference to the definitions of set equality, subset relation, intersection, union, complement, where appropriate.
   i. *Let A, B be sets. Suppose $A \subset A \backslash B$. Then $A \cap B = \emptyset$.*
   ii. *Let A, B be sets. Suppose $A \cap B = \emptyset$. Then $A \subset A \backslash B$.*

4. We introduce/recall the definition of *column space* for matrices (with real entries):

   Let A be an $(m \times n)$-matrix with real entries.

   The column space of A is defined to be the set $\left\{ \mathbf{y} \in \mathbb{R}^m : \begin{array}{l} \text{There exists some } \mathbf{x} \in \mathbb{R}^n \\ \text{such that } \mathbf{y} = A\mathbf{x} \end{array} \right\}$.

   It is denoted by $\mathcal{C}(A)$.

   Prove the statements below, with reference to the definition of *column space*:

   (a) *Suppose B is an $(n \times n)$-square matrix with real entries. Then $\mathcal{C}(B^2) \subset \mathcal{C}(B)$.*

   (b) *Let D be an $(m \times n)$-matrix with real entries, and E, F be $(n \times p)$-matrices with real entries.*
   Suppose $\mathcal{C}(E) \subset \mathcal{C}(F)$. Then $\mathcal{C}(DE) \subset \mathcal{C}(DF)$.