MATH1050 Proof-writing Exercise 4 Index of Comments

1. **CR.**

   You are cramming two or more ideas into the same line, thus making the whole line unclear.

   Examples.

   (a) It is unclear what is meant by

   ‘*Let* $u = x + y \in \mathbb{Z}$.’

   ‘*Let* $u = x + y$’ is one thing: you are defining u to be the sum $x + y$.

   ‘$x + y \in \mathbb{Z}$’ is another thing: you are arguing that $x + y$ is an integer because of so-and-so.

   (b) It is unclear what is meant by

   - ‘*Suppose it were true that* $p + q \geq (s + t)^2 \geq 0$.’

   ‘*Suppose it were true that* $p+q \geq (s+t)^2$’ is one thing: you are stating some assumption which you probably intend to use in the subsequent argument.

   ‘$(s + t)^2 \geq 0$’ is another thing: you are arguing that $(s + t)^2$ is a non-negative real number because of so-and-so.

   You should slow down, and write out the ideas one-by-one, expounding at most one idea within one sentence.

2. **FOR.**

   There are too many meanings for the word ‘*for*’. Choose an appropriate word, other than ‘*for*’ to indicate what you actually means. (You do not want the reader to read your passage in such a way that you don’t intend.)

3. **IA.**

   This is redundant and confusing use of the quantifier ‘for any’.

   You have already introduced, say, a concrete object a, earlier. After that point, it is wrong to write ‘*for any* a *...*’, because it would confuse the reader on whether you are talking about the same a, or you want to introduce another object which you quite careless and wrongly label as a. In fact, if you are doing things correctly, you can be (as you should be) referring to the same object a.

4. **IP.**

   Pay attention when dealing with the inequality symbols ‘$\geq$’, ‘$\leq$’, ‘$>$’, ‘$<$’, and the terms ‘*positive*’, ‘*non-negative*’, ‘*negative*’, ‘*non-positive*’ in a mathematical text.

   ‘$\geq$’ is the symbol for ‘*greater than or equal to*’. ‘$>$’ is the short-hand for ‘*(strictly) greater than*’.

   (a) ‘$a \geq b$’ reads ‘a *is greater than or equal to* b’; very formally but accurately presented (with the role of the disjunction ‘*or*’ highlighted), ‘$a \geq b$’ reads ‘a is (strictly) greater than b or a is equal to b’.

   For this reason, the negation of ‘$a \geq b$’ is ‘$a < b$’.

   It is wrong to use ‘$a \leq b$’ as the negation of ‘$a \geq b$’.

   (b) ‘$a \geq 0$’ reads ‘a *is greater than or equal to* 0’, or in a more colloquial way, ‘a *is non-negative*’; very formally presented, ‘$a \geq 0$’ reads ‘a *is positive or* a *is equal to* 0.’

   ‘$a > 0$’ reads ‘a *is (strictly) positive*’.

   It is wrong to read ‘$a \geq 0$’ as ‘a *is positive*’.

   (c) The square of a real number is non-negative; it is not guaranteed to be positive, unless it is known that it is non-zero. It can happen that the square of a real number is non-positive: it happens exactly when the number concerned is zero.

   Et cetera.

   Always remember: it is wrong to read $a \leq b$ as

   ‘a is less than and equal to b.’

   It is dangerous and misleading to read $a \leq b$ as

   ‘a is “less than, equal to” b.’

   because it would be thus be one small step from the mental slip of mis-read ‘$\leq$’ as ‘*less than and equal to*’.

5. **LU.**

   The logic in this passage is unclear.

6. **MA.**

   At least part of the assumptions is missing. But you are going to use these assumptions in the argument. The reader is not responsible to write out the missing assumptions for you, and will simply regard your argument as wrong when you are applying the 'missing assumptions'.

   In a proof-by-contradiction argument, it is totally unacceptable to omit any part of the assumption used for obtaining the contradiction.

7. **MEAN.**

   What is '*this means*'? What is '*which means*'? The logic in unclear. Do not use the word '*mean*' in a formal argument. Find a more appropriate word whenever you want to use '*mean*'.

8. **MS.**

   There is a missing step which you should not have skipped.

9. **PC.**

   Punctuation and capitals should be used appropriately so as to indicate to the reader how the passage is to be read. Omissions may result in the reader being confused with the logic and/or the mathematical content in what you are writing.

10. **SISU.**

    '*Since*' (or '*because*') is different from '*suppose*' (or 'assume'). Look up the entries in the dictionary. Do not confuse these words when you read and/or write.

    When you write

    > '*Suppose H holds. Then K holds.*'

    you are suggesting to the reader that the statement H is valid from this point on only because you are supposing it to be true. You are 'restricting' yourself to the 'special' situation in which H is true (whereas in general H is not necessarily true).

    When you write

    > '*Because H holds, (then) K holds.*'

    you are suggesting to the reader that the statement H is valid prior to this point. You are now only invoking its validity. H is now valid because you have supposed it to be true at some point earlier in the argument, or it is known to be true irrespective of what you are doing in this argument.

    Also, think carefully, in the middle of an argument, whether you mean 'if ... then ...' or 'because ..., (therefore) ...'. They are not interchangeable.

11. **WD.**

    The deduction at this place is wrong.