

MATH1050 Proof-writing Exercise 3 Index of Comments

1. CASE.

You should indicate clearly to the reader that you are ‘splitting’ the argument into various cases, in each of which there may be specific extra assumptions.

2. DEF.

Look up the definition. (In this case, it is the definition of divisibility for integers.) You are not adhering to definition in your argument, or you have missed out key logical features in the definition. For this reason, your argument is deemed wrong.

3. DP.

Dependence amongst various objects must be made clear. This is especially the case when you want to introduce new objects out of old ones (which are already present).

For example, given that z is an Gaussian integer and that z is already present in the discussion, when you want to introduce its real and imaginary parts and further quote from definition that these are integers, it is acceptable to write:

‘ $z = \operatorname{Re}(z) + i\operatorname{Im}(z)$. By the definition of Gaussian integers, $\operatorname{Re}(z)$, $\operatorname{Im}(z)$ are integers.’

The symbols $\operatorname{Re}(z)$, $\operatorname{Im}(z)$ indicate the dependence of these two real numbers on z .

It is also correct to write:

‘There exist some $s, t \in \mathbb{Z}$ such that $z = s + ti$.’

The dependence of s, t on z is embedded into this ‘existence statement’.

- It is wrong to just write something like:

‘ $z = s + ti$, $s, t \in \mathbb{Z}$.’

The reason is that the dependence is not made clear.

- It is inappropriate to write something like:

‘ $z = s + ti$ where $s, t \in \mathbb{Z}$.’

The reason is the meaning of the word ‘where’ is unclear.

4. FOR.

There are too many meanings for the word ‘for’. Choose an appropriate word, other than ‘for’ to indicate what you actually means. (You do not want the reader to read your passage in such a way that you don’t intend.)

5. IE.

This is redundant and confusing use of the quantifiers ‘there exist’, ‘for some’.

You have already introduced, say, a concrete object c , earlier. After that point, it is wrong to write ‘for some c ...’, ‘there exists c ...’, because it would confuse the reader on whether you are talking about the same c , or you want to introduce another object which you quite careless and wrongly label as c .

6. LW.

The logic in this passage is wrong.

7. MA.

At least part of the assumptions is missing. But you are going to use these assumptions in the argument. The reader is not responsible to write out the missing assumptions for you, and will simply regard your argument as wrong when you are applying the ‘missing assumptions’.

8. MC.

There is a missing case. Your subsequent argument is therefore incomplete and/or wrong.

9. MEAN.

What is ‘this means’? What is ‘which means’? The logic is unclear. Do not use the word ‘mean’ in a formal argument. Find a more appropriate word whenever you want to use ‘mean’.

10. NZ.

You are dividing by something which is not known to be zero or not.

11. **PC.**

Punctuation and capitals should be used appropriately so as to indicate to the reader how the passage is to be read. Omissions may result in the reader being confused with the logic and/or the mathematical content in what you are writing.

12. **QF.**

At least one of the things described below has happened:

(a) You have misunderstood the logical structure and the mathematical content in

- i. '*there exists some $k \in \mathbb{Z}$ such that $a = kb$* '
- ii. ' *$u = sv$ for some Gaussian integer s* '.

Be aware of the information '*which depends on which*' carried in such a statement. (In the first example, k depends on a, b , and not the other way round. In the second example, s depends on u, v , and not other way round.)

(b) You have missed the key idea of 'existence' in

- i. '*there exists some $k \in \mathbb{Z}$ such that $a = kb$* '
- ii. ' *$u = sv$ for some Gaussian integer s* '.

This matters in the logical structure of the argument. Your subsequent argument fails entirely because of it. (In the first example, k owes its existence from a, b in the context of the argument. In the second example, s owes its existence from u, v in the context of the argument.)

(c) This statement in which the existential quantifier '*there exist*' (or '*for some*') has appeared is not stated in an appropriate way: its logical structure has not been given due respect.

It is tempting to use '*where*' as a substitute for '*for some*'. The problem is that this leads to various for mis-interpretation (including the misreading of such a '*where*' as '*for any*'.)

13. **SISU.**

'*Since*' (or '*because*') is different from '*suppose*' (or '*assume*'). Look up the entries in the dictionary. Do not confuse these words when you read and/or write.

Also, think carefully, in the middle of an argument, whether you mean 'if ... then ...' or 'because ..., (therefore) ...'. They are not interchangeable.

14. **WD.**

The deduction at this place is wrong.