MATH1050 Proof-writing Exercise 3

Advice.

- Remember to adhere to definition, always.
- See the feedback to your work on Proof-writing Exercise 1, when it becomes available.

1. We introduce the definitions below:
   - *Let $z \in \mathbb{C}$. The number z is said to be a* **Gaussian integer** *if both of $\mathsf{Re}(z)$, $\mathsf{Im}(z)$ are integers.*
   - *The set of all Gaussian integers is denoted by $\mathbb{G}$.*

   Prove the statements below:

   (a) *Suppose s is an integer. Then s is a Gaussian integer.*

   (b) *Suppose s is an integer. Then si is a Gaussian integer.*

   (c) *Let s is a Gaussian integer. Suppose $s \neq 0$. Then $|s| \geq 1$.*

   (d) *Suppose s, t are Gaussian integers. Then $\bar{s}$, $s + t$ and st are Gaussian integers.*

2. We introduce the definition below:
   - *Let $u, v \in \mathbb{G}$. The number u is said to be* **$\mathbb{G}$-divisible** *by v if there exists some $s \in \mathbb{G}$ such that $u = sv$.*

   Prove the statements below:

   (a) $25i$ *is $\mathbb{G}$-divisible by* $3 + 4i$.

   (b) 0 *is $\mathbb{G}$-divisible by* 0.

   (c) *Let $u, v \in \mathbb{G}$. Suppose $u \neq 0$ and u is $\mathbb{G}$-divisible by v. Then $|v| \leq |u|$.*

   (d) *Let $u, v, w \in \mathbb{G}$. Suppose (u is $\mathbb{G}$-divisible by v and v is $\mathbb{G}$-divisible by w). Then u is $\mathbb{G}$-divisible by w.*