

## MATH1050 Proof-writing Exercise 1 (Answers and selected solution)

### 1. Answer.

Let  $u, v$  be integers. The integer  $u$  is said to be divisible by the integer  $v$  if there exists some integer  $k$  such that  $u = kv$ .

### 2. Solution.

(a)  $0 = 0 \cdot 0$ .

0 is an integer.

Therefore 0 is divisible by 0.

(b) Let  $x$  be an integer. Suppose  $x$  is divisible by 0.

Then, by definition, there exists some integer  $k$  such that  $x = k \cdot 0$ .

Therefore (for the same  $k$ ), we have  $x = k \cdot 0 = 0$ .

### 3. Solution.

Let  $x, y \in \mathbb{Z}$ . Suppose  $x$  is divisible by  $y$  and  $y$  is divisible by  $x$ .

Since  $x$  is divisible by  $y$ , there exists some  $k \in \mathbb{Z}$  such that  $x = ky$ .

Since  $y$  is divisible by  $x$ , there exists some  $\ell \in \mathbb{Z}$  such that  $y = \ell x$ .

We have  $x = ky = k\ell x$ .

Then  $(k\ell - 1)x = 0$ .

Therefore  $x = 0$  or  $k\ell = 1$ .

- (Case 1). Suppose  $x = 0$ . Then  $y = 0$ . Therefore  $|x| = |y|$ .
- (Case 2). Suppose  $x \neq 0$ . Then  $1 = k\ell$ . Since  $k, \ell$  are integers, we have  $k = \ell = 1$  or  $k = \ell = -1$ . Then  $|x| = |ky| = |y|$ .

Hence in any case,  $|x| = |y|$ .