

MATH1050 Exercise 8 Supplement (Answers)

1. (a) —
(b) 0 is a lower bound of $\{b_n\}_{n=0}^{\infty}$
2. —
3. —
4. (a) 2 is the greatest element of S .
(b) —
(c) S is bounded below by 1 in \mathbb{R} .
5. (a) 2 is the greatest element of S .
(b) S has no least element.
(c) S is bounded below by 0 in \mathbb{R} .
6. —
7. —
8. —
9. —
10. (a) *Hint.* When f is not ‘identically zero’ on $[a, b]$, study the quadratic function $F : \mathbb{R} \rightarrow \mathbb{R}$ defined by $F(t) = \int_a^b (tf(u) + g(u))^2 du$ for any $t \in \mathbb{R}$.
(b) —
11. —
12. (a) —
(b) —
(c) *Hint.* Write $A = \int_0^{\frac{1}{2}} (f'(t))^2 dt$, $B = \int_{\frac{1}{2}}^1 (f'(t))^2 dt$. By the result in part (b.i), we have $(f(x))^2 \leq Ax$ for any $x \in [0, \frac{1}{2}]$. By the result in part (b.ii), we have $(f(x))^2 \leq B(1 - x)$ for any $x \in [\frac{1}{2}, 1]$. So what happens?
13. —
14. —
15. —
16. —
17. —
18. —
19. —
20. (a) —
(b) i. $R = 1, S = 1, T = 1$.
ii. —
iii. —
iv. —
(c) —

Remark. *Beyond MATH1050 and towards mathematical analysis.* According to the Bounded-Monotone Theorem and the Sandwich Theorem, for each $\alpha > 0$, the infinite sequences $\{a_n(\alpha)\}_{n=2}^{\infty}$, $\{b_n(\alpha)\}_{n=2}^{\infty}$, $\{c_n(\alpha)\}_{n=2}^{\infty}$ defined by

$$a_n = \left(1 + \frac{\alpha}{n}\right)^n, \quad b_n = \sum_{k=0}^n \frac{\alpha^k}{k!}, \quad c_n = \left(1 - \frac{\alpha^2}{2n}\right) \sum_{k=0}^n \frac{\alpha^k}{k!}, \quad \text{whenever } n \geq 2$$

‘converges’ to the same limit. This limit is the number e^α .

21. —
22. —
23. —