MATH1050 Exercise 4 Supplement (Answers)

1. ——
2. ——
3. ——
4. ——
5. ——
6. ——
7. (a) ——

   (b) $\sum_{k=1}^{2m} \cos^2(k\beta) = \dfrac{\cos(2m\beta)\sin((2m+1)\beta)}{2\sin(\beta)} + \dfrac{2m-1}{2}$ and $\sum_{k=1}^{2m} \sin^2(k\beta) = \dfrac{\cos(2m\beta)\sin((2m+1)\beta)}{2\sin(\beta)} + \dfrac{2m+1}{2}$
8. ——
9. ——
10. ——
11. ——
12. **Remark.** Begin the argument with this assumption: '*Let $\{a_n\}_{n=1}^{\infty}$ be an infinite sequence in $\mathbb{N}$. Suppose $n \leq \sum_{j=1}^{n} {a_j}^2 \leq n + 1 + (-1)^n$ for each positive integer n.*' Now apply mathematical induction to the proposition $P(n)$ below:
    - $a_1 = a_2 = \cdots = a_n = 1$.
13. (a) **Remark.** Apply mathematical induction to the proposition $P(n)$ below:
       - $(\sqrt{3}+1)^{2n+1} - (\sqrt{3}-1)^{2n+1}$ *is an integer which is divisible by* 2^{n+1}, *and* $(\sqrt{3}+1)^{2n+3} - (\sqrt{3}-1)^{2n+3}$ *is an integer which is divisible by* 2^{n+2}.

    (b) **Remark.** Apply mathematical induction to the proposition $P(n)$ below:
       - $(3+\sqrt{5})^{n+1} + (3-\sqrt{5})^{n+1}$ *is an integer which is divisible by* 2^{n+1}, *and* $(3+\sqrt{5})^{n+2} + (3-\sqrt{5})^{n+2}$ *is an integer which is divisible by* 2^{n+2}.
14. ——
15. ——
16. ——
17. ——
18. (a) **Remark.** Apply mathematical to the proposition $P(n)$ below:
       - *Suppose $z_1, z_2, \cdots, z_n$ are complex numbers. Then* $\sqrt{|z_1|^2 + |z_2|^2 + \cdots + |z_n|^2} \leq |z_1| + |z_2| + \cdots + |z_n|$.

    (b) **Remark.** Apply mathematical to the proposition $P(n)$ below:
       - *Suppose $\theta_1, \theta_2, \cdots, \theta_n \in (0, \pi)$. Then* $|\sin(\theta_1 + \theta_2 + \cdots + \theta_n)| < \sin(\theta_1) + \sin(\theta_2) + \cdots + \sin(\theta_n)$.

    (c) **Remark.** Apply mathematical to the propostion $P(n)$ below:
       - *Suppose $a_1, a_2, \cdots, a_n$ are positive real numbers. Then* $(a_1 + a_2 + \cdots + a_n)\left(\dfrac{1}{a_1} + \dfrac{1}{a_2} + \cdots + \dfrac{1}{a_n}\right) \geq n^2$.

    (d) **Remark.** Begin the argument with this assumption: '*Let $s, t \in \mathbb{Q}$, with $t > 0$.*' From this point on, s, t are fixed. Now apply mathematical to the propostion $P(n)$ below:
       - *There exist $a, b \in \mathbb{Q}$ such that* $(s + \sqrt{t})^n = a + b\sqrt{t}$.
19. ——
20. ——
21. (a) i. $A = B = C = \dfrac{1}{2}$.

ii.

(b) i. $M = 243$, $N = 1$.

ii. $r = 3$.

22.