MATH 2050A - HW 3 - Comments and Common Mistakes:

Common Misktakes:

Question 1:

Using the method in Example 2, Sec. 19, show that $f'(z)$ does not exist at any $z \in \mathbb{C}$ if

a) $f(z) = \text{Re}(z)$ b) $f(z) = \text{Im}(z)$

1. The requirement that f' does not exists *for all* $z \in \mathbb{C}$ is crucial. (I believe you must have encountered functions that are not complex differentiable at every point on the domain it is defined on.) Forgetting to emphasize that derivatives does not exist *for all* $z \in \mathbb{C}$ will result in mark deduction.

2. A number of you have shown the non-existence of limit by saying something like:

   Consider only the $\frac{\bar{w}}{2w}$ part:
   1° If w is a real number, then $\frac{\bar{w}}{2w} = \frac{1}{2}$
   2° If w is a pure imaginary number, then $\frac{\bar{w}}{2w} = -\frac{1}{2}$
   Hence we know that for any $z \in \mathbb{C}$, $f'(z)$ doesn't exist.

   Although NO mark was deducted, I would not recommend writing this way. You are reminded that limits concern behaviours at infinity (along certain directions if it's the case of directional derivatives). Please at least write something like when w *approaching* to 0 *along* the real-axis or when w *approaching* to 0 *along* the pure-imaginary axis, but not simply stating what happens when w is real or pure imaginary. "Approach" and "along" here are the key words.

3. As a remark, an explicit way to illustrate the non-existence of limit (in $\mathbb{C}$) is to show via sequences and make use of the sequential criteria for limits (in $\mathbb{C}$). Please refer to my solution for details.

Question 2:

Using theorems in Sec. 21 and 23, for each of the following functions f on $\mathbb{C}$, determine where $f'(z)$ exists and find the corresponding value for such z.

a) $f(z) = \frac{1}{z}$ b) $f(z) = x^2 + iy^2$ c) $f(z) = z \,\text{Im}\, z$

where x, y denote $\text{Re}\, z, \text{Im}\, z$ respectively.

1. This question requires you to determine the differentiable points of the given function. You have to show a certain set of points are differentiable points *precisely*: you have to show why other points are *not* differentiable points. Missing any direction will result in mark deduction.

2. It is a standard common mistake that students mis-used the Cauchy-Riemann Equations as sufficiency for complex differentiability: you have to show partial derivatives to be continuous at the point and exist in a neighborhood of the point!

3. *Many* marks will be lost if you missed a number of the above. **Nearly all of you have some marks deducted in this question.** Indeed, if you miss to state the continuity condition for using CR equations as sufficiency throughout your HW, at least 2.5 marks will be deducted. As a matter of fact, the median of this HW is 7.5.

Question 3:

Let $f(z) = exp(z^2)$.

(i). Give two proofs that the function f is entire, that is, complex differentiable (or holomorphic) on all of $\mathbb{C}$.

(ii). Find the derivative of f.

1. The two most used proofs are those using the Cauchy Riemann Equations and those using the chain rule. (A few students prove by computing the limit of difference quotient directly.)

2. The former has the same problem with the previous question: only the Cauchy-Riemann equations are considered, but the continuity (and the existence) of partial derivarives is missed.

3. While proving using the Chain Rule is a lot easier, I demand you to state clearly the Chain Rule has been used instead of writing simply compositions of differentiable functions are differentiate. At this stage, you should be seeing that complex differentiability is quite different from real differentiability. It is not trivial for the chain rule to also be applicable for the complex case. As a rule of thumb, I expect any use of theorems should be stated clearly.

General Comments:

1. The graded problems are Q1, 5 and 7. Q1 carries 2 marks; Q5 carries 4 marks; Q7 carries 3 marks; 1 mark is for presentation and effort. Full solutions are provided for these problems while only partial solutions are provided for other problems.

2. I feel that many of you did not pay attention to details of the condition in theorems (and problems), for example, the continuity of partial derivatives is often missed in the sufficiency of Cauchy Riemann Equations to holomorphy. You are recommended to read related section of the lecture note before doing HW. That helps you understand the theorems (your tools) better so that you can apply them freely and correctly.