Assignment 6

Coverage: 15.8 in Text.

Exercises: 15.8 no 5, 7, 9, 13, 15, 19, 20, 25.

Additional and Advanced Exercises: 12, 14, 19, 23.

Submit 15.8 no. 15, 20; Additional and Advanced Ex. no. 12, 14 by March 2.

Supplementary Problems

1. Find the volume of the ball in $\mathbb{R}^4$, that is, $\{(x, y, z, w) : x^2+y^2+z^2+w^2 \leq R^2\}$. Suggestion: Apply the change of variables formula after introducing generalized polar coordinates $w = \rho\cos\psi, z = \rho\sin\psi\cos\varphi, x = \rho\sin\psi\sin\varphi\cos\theta, y = \rho\sin\psi\sin\varphi\sin\theta,\ \psi, \varphi \in [0, \pi],\ \theta \in [0, 2\pi]$.

Assignment 6

Please submit the following questions by **2 Mar 2021, 23:00**.

§15.8: Q15, Q20
Additional and Advanced Exercises (§AA, for simplicity): Q12, Q14.

§15.8 Q15

Use the transformation $x = u/v$ and $y = uv$ to evaluate the integral sum

$$\int_1^2 \int_{1/y}^{y} (x^2 + y^2)\, dx\, dy + \int_2^4 \int_{y/4}^{4/y} (x^2 + y^2)\, dx\, dy.$$

§15.8 Q20

Let D be the region in xyz-space defined by the inequalities

$$1 \le x \le 2, \quad 0 \le xy \le 2, \quad 0 \le z \le 1.$$

Evaluate

$$\iiint_D (x^2 y + 3xyz)\, dx\, dy\, dz$$

by applying the transformation

$$u = x, \quad v = xy, \quad w = 3z$$

and integrating over an appropriate region G in uvw-space.

§AA Q12

(a) **Polar coordinates** Show, by changing to polar coordinates that

$$\int_0^{a \sin\beta} \int_{y \cot\beta}^{\sqrt{a^2 - y^2}} \ln(x^2 + y^2)\, dx\, dy = a^2 \beta \left(\ln a - \frac{1}{2} \right)$$

where $a > 0$ and $0 < \beta < \pi/2$.

(b) Rewrite the Cartesian integral with the order of integration reversed.

§AA Q14

Transforming a double integral to obtain constant limits
Sometimes a multiple integral with variables limits can be changed into one with constant limits. By changing the order of integration, show that

$$\begin{aligned}\int_0^1 f(x)\left(\int_0^x g(x-y)f(y)\,dy\right)dx &= \int_0^1 f(y)\left(\int_y^1 g(x-y)f(x)\,dx\right)dy\\ &= \frac{1}{2}\int_0^1\int_0^1 g(|x-y|)f(x)f(y)\,dx\,dy\end{aligned}$$