MATH2010 Advanced Calculus I, 2020-21

HOMEWORK THREE

Due 3pm Monday, Nov. 23

Q1. A function $f(x, y)$ is *homogeneous of degree* n (n a nonnegative integer) if $f(tx, ty) = t^n f(x, y)$ for all t, x, and y. For such a smooth function, prove that

(a) $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf(x, y)$.

(b) $x^2(\frac{\partial^2 f}{\partial x^2}) + 2xy(\frac{\partial^2 f}{\partial x \partial y}) + y^2(\frac{\partial^2 f}{\partial y^2}) = n(n-1)f$.

Q2. Show that the curve

$$\vec{r}(t) = \langle \ln t, t \ln t, t \rangle$$

is tangent to the surface

$$xz^2 - yz + \cos xy = 1$$

at $(0, 0, 1)$.

Q3. Find the absolution maxima and minima of the functions on the given domains.

(a) $f(x, y) = x^2 + xy + y^2 - 6x$ on the rectangular plate $0 \leq x \leq 5$, $-3 \leq y \leq 3$.

(b) $f(x, y) = 6xye^{-(2x+3y)}$ in the closed first quadrant (includes the non-negative axes).

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