

MATH2010 Advanced Calculus I, 2020-21

HOMEWORK ONE

Due 3pm Monday, Oct. 12

- Q1.** Find the angle between the diagonal of a cube and one of the edges it meets. In particular, is it greater or less than $\pi/4$?
- Q2.** (a) Use vectors to show that the distance from $P_1 = (x_1, y_1, z_1)$ to the plane $Ax + By + Cz = D$ is

$$d = \frac{|Ax_1 + By_1 + Cz_1 - D|}{\sqrt{A^2 + B^2 + C^2}}$$

- (b) Find an equation for the sphere that is tangent to the planes $x + y + z = 3$ and $x + y + z = 9$ if the planes $2x - y = 0$ and $3x - z = 0$ pass through the center of the sphere.
- Q3.** (a) At what times in the interval $0 \leq t \leq \pi$ are the velocity and acceleration vectors of the motion $\vec{r}(t) = \mathbf{i} + (5 \cos t)\mathbf{j} + (3 \sin t)\mathbf{k}$ orthogonal?
- (b) Write an integral for the total distance travelled in the time interval $0 \leq t \leq \pi$. (Do not try to evaluate the integral.)
- Q4.** Evaluate the following limits or show they do not exist.

- (a) $\lim_{(x,y) \rightarrow (2,-4)} \frac{y+4}{x^2y - xy + 4x^2 - 4x}$
- (b) $\lim_{(x,y) \rightarrow (0,0)} (2x^2 + y^2) \sin \frac{1}{\sqrt{x^2 + 4y^2}}$.
- (c) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^5y^2}{x^{10} - y^4}$.
- (d) $\lim_{(x,y) \rightarrow (1,-1)} \frac{xy + 1}{x^2 - y^2}$
- (e) $\lim_{(x,y) \rightarrow (0,0)} \frac{2x}{x^2 + x + y^2}$

—END—