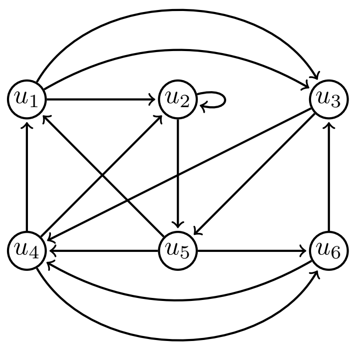
MMAT5380 Graph Theory and Networks
Suggested Solution for Assignment 2

2-1:

$A =$

	u_1	u_2	u_3	u_4	u_5	u_6	sum = $\deg^+$
u_1	0	1	2	0	0	0	3
u_2	0	1	0	0	1	0	2
u_3	0	0	0	1	1	0	2
u_4	1	1	0	0	0	1	3
u_5	1	0	0	1	0	1	3
u_6	0	0	1	1	0	0	2
sum = $\deg^-$	2	3	3	3	2	2	



2-2: (a) $e(v_1) = 3$, $e(v_2) = 3$, $e(v_3) = 2$, $e(v_4) = 2$, $e(v_5) = 2$, $e(v_6) = 2$, $e(v_7) = 3$.

(b) v_3, v_4, v_5, v_6 are centers.

(c) The radius is 2 and the diameter is 3.

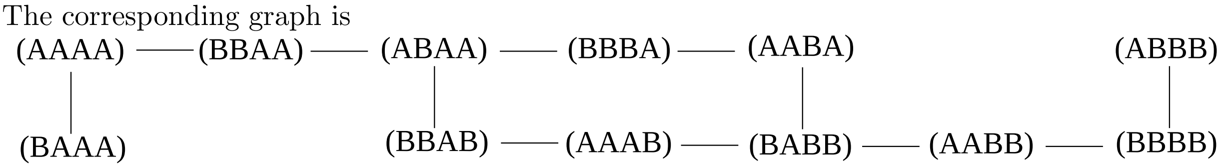
(d) $v_1v_3v_4v_2v_5v_6v_7$ is a longest path (or $v_2v_4v_3v_1v_5v_6v_7$, $v_4v_3v_1v_2v_5v_6v_7$).

2-3: (a) Choose five (a, f)-walks from $ababf$, $abadf$, $abcbf$, $abebf$, $abedf$, $abfbf$, $abfdf$, $adabf$, $adadf$, $adebf$, $adedf$, $adfbf$, $adfdf$, $aeabf$, $aeadf$ and $aecbf$.

(b) There are $abceadf$, $aebcedf$, $aecbadf$ and $aecbedf$.

(c) There are $abcedf$ and $adecbf$.

2-4: The corresponding graph is



There are two shortest ways for the man to cross the river. They are

(AAAA)-(BBAA)-(ABAA)-(BBBA)-(AABA)-(BABB)-(AABB)-(BBBB) and
(AAAA)-(BBAA)-(ABAA)-(BBAB)-(AAAB)-(BABB)-(AABB)-(BBBB).

Note that (ABBA), (BAAB), (ABAB) and (BABA) are not allowable.

2-5: Obviously, statements $(a), (b)$ and (c) are true by the definition of distance. For (d), let P_1 be the path from u to v with length $d(u, v)$ and P_2 be the path from v to w of length $d(v, w)$. Then P_1P_2 is a (u, w)-walk with length $d(u, v) + d(v, w)$. By Lemma 2.1.3, there is a (u, w)-path P in P_1P_2. By the definition of distance, $d(u, w)$ is the least length of paths from u to w. Thus $d(u, v) + d(v, w) \geq d(u, w)$.

2-6: Let $P = u_0u_1 \cdots u_k$ be a longest path of G. Since P is a longest path, all neighbors of u_0 lie in P. Since $\delta \geq 2$, there is another edge is incident with u_0 but it is not in P, say u_0u_l, where $l \geq 0$. Hence $u_0 \cdots u_lu_0$ is a cycle.

Note that l can be 0. For this case, there is a loop incident with u_0.

2-7: We know that $2q = \sum_{v \in V(G)} \deg(v) \geq p\delta$ which implies that $\delta \leq \frac{2q}{p}$. By Theorem 2.4.6, $\kappa(G) \leq \delta(G)$ we have $\kappa(G) \leq \frac{2q}{p}$. Note that $\kappa(G)$ is an integer. Therefore $\kappa(G) \leq \lfloor \frac{2q}{p} \rfloor$.