MMAT5320 Computational Mathematics

Assignment 2

Due date: 19th November 2019

Please hand in your assignments during the class on **Tuesday 19th November 2019**. Remember to include your name, ID number and show all of your working! **Points will be deducted for incorrect working even if the final answer is correct**.

Question 1

(i) [12 pts] Apply the Gram–Schmidt orthonormalisation process to the following set of vectors

$$\vec{a}_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \vec{a}_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \quad \vec{a}_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix},$$

and write down the reduced QR factorization for the matrix A with columns $\vec{a}_1, \ldots, \vec{a}_3$.

(ii) [13 pts] Apply the Gram–Schmidt projection method to the following set of vectors

$$\vec{b}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \vec{b}_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \quad \vec{b}_3 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix},$$

and write down the reduced QR factorization for the matrix B with columns $\vec{b}_1, \vec{b}_2, \vec{b}_3$.

Question 2

(i) [7 pts] For $\vec{x} \in \mathbb{R}^2$, let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be an orthogonal matrix such that $F\vec{x} = \begin{pmatrix} \|\vec{x}\| \\ 0 \end{pmatrix}$. Explain how to derive the formula $F = I - 2\frac{\vec{v}\vec{v}^\top}{\|\vec{v}\|_2^2}$, with $\vec{v} = F\vec{x} - \vec{x}$ using the figure from the lecture slide 65 titled "Householder reflectors"

(ii) [16 pts] Apply Householder's method to obtain a full QR factorization of the matrix

$$A = \begin{pmatrix} -1 & 0 & 1 \\ 1 & 1 & 1 \\ -1 & -1 & 1 \\ 1 & 0 & 1 \end{pmatrix}.$$

(iii) [4 pts] Compute the pseudoinverse for A and solve the overdetermined problem

$$A\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 1 \\ 0 \end{pmatrix}.$$

Question 3

(i) [7 pts] Investigate whether the following matrices are unitary diagonalisable

$$A_1 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}.$$

(ii) [9 pts] Find a Schur factorisation of the matrix

$$A = \begin{pmatrix} 7 & -2 \\ 12 & -3 \end{pmatrix}.$$

Question 4

In the following please give your answers either as fractions or to 4 decimal points.

(i) [6 pts] Provide the first two steps of the Power iteration in order to calculate the eigenvalues of the matrix

$$A = \begin{pmatrix} -3 & 6 \\ -1 & 11 \end{pmatrix}$$

with initial vector $(1,0)^\top$.

(ii) [6 pts] Show the first step of the shifted inverse iteration applied to find an eigenvalue of

$$A = \begin{pmatrix} -3 & 6 \\ -1 & 11 \end{pmatrix}$$

with initial vector $(1,0)^\top$ and shift $\mu = 2$.

(iii) [4 pts] Compute the first iteration of the Rayleigh quotient iteration for the matrix

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

with initial vector $x_0 = (1,1)^\top/\sqrt{2}$. Include in your answer the normalised approximate eigenvectors x_1, and the Rayleigh quotient r_1.

(iv) [9 pts] Perform one iteration of the unshifted QR algorithm to the matrix

$$A = \begin{pmatrix} 3 & 1 \\ 4 & -1 \end{pmatrix}.$$

You may find it helpful to first find a QR factorisation of A.