1. Let \(C\) be a nonempty convex subset of \(\mathbb{R}^n\). Let \(f = (f_1, ..., f_m)\), where \(f_i : C \to \mathbb{R}\), \(i = 1, ..., m\), are convex functions, and let \(g : \mathbb{R}^m \to \mathbb{R}\) be a convex function such that \(g(u_1) \leq g(u_2)\), for all \(u_1 \leq u_2\) in a convex set that contains \(\{ f(x) | x \in C \}\). Show that \(h\) defined by \(h(x) = g(f(x))\) is convex over \(C\). If in addition, \(m = 1\), \(g\) is strictly increasing and \(f\) is strictly convex, show that \(h\) is also strictly convex.

2. Show that the following functions are convex:
   (a) \(f_1(x) = \ln(e^{x_1} + ... + e^{x_n})\)
   (b) \(f_2(x) = ||x||^p\) with \(p \geq 1\)
   (c) \(f_3(x) = e^{x^T A x}\), where \(A\) is a positive semidefinite symmetric \(n \times n\) matrix

3. Let \(f : \mathbb{R}^n \to \mathbb{R}\) be a differentiable function. We say that \(f\) is strongly convex with coefficient \(\alpha\) if
   \[
   (\nabla f(x) - \nabla f(y))^T (x - y) \geq \alpha \|x - y\|^2, \forall x, y \in \mathbb{R}^n,
   \]
   where \(\alpha\) is some positive scalar.
   (a) Show that if \(f\) is strongly convex with coefficient \(\alpha\), then \(f\) is strictly convex.
   (b) Assume that \(f\) is twice continuously differentiable. Show that strongly convexity of \(f\) with coefficient \(\alpha\) is equivalent to the positive semi definiteness of \(\nabla^2 f(x) - \alpha I\) for every \(x \in \mathbb{R}^n\), where \(I\) is the identity matrix.

4. We say that \(f : \mathbb{R}^n \to \mathbb{R}\) is positively homogeneous if \(f(\alpha x) = \alpha f(x)\) for all \(\alpha > 0\), and that \(f\) is subadditive if \(f(x + y) \leq f(x) + f(y)\) for all \(x, y \in \mathbb{R}^n\). Show that a positively homogeneous function is convex if and only if it is subadditive.