1. (a) We may assume $\lambda_2 > 0$ ($\lambda_2 > 0$ is easy). Let $x \in C$. Since $0 \in C$, $\lambda_1 x + \lambda_2^{-1} \lambda_1 x + \lambda_2^{-1} \lambda_1 x = 0 \in C$.

Then $\lambda_1 x = \lambda_2 (\frac{\lambda_1}{\lambda_2} x) \in \lambda_2 C$.

(b) We may assume $\alpha + \beta > 0$ ($\alpha + \beta = 0$ is easy). Then

$$\alpha x + \beta y = (\alpha + \beta)\left(\frac{\alpha}{\alpha + \beta} x + \frac{\beta}{\alpha + \beta} y\right)$$

2. Suppose $\sum_{i=1}^m \lambda_i x_i + \lambda x = 0$ and $\sum_{i=1}^m \lambda_i + \lambda = 0$.

Suppose $\lambda \neq 0$. Then

$$-\sum_{i=1}^m \frac{\lambda_i}{\lambda} = 1$$

So $x = -\sum_{i=1}^m \frac{\lambda_i}{\lambda} x_i \in \text{aff}\{x_1, ..., x_m\}$, which is a contradiction.

Hence $\lambda = 0$. Then $\lambda_i = 0$ since $x_1, ..., x_m$ are affinely independent.

Therefore, $x_1, ..., x_m, x$ are affinely independent.

3. Since $\{x_0, ..., x_m\} \subset \Delta_m \subseteq C$, $\text{aff}\{x_0, ..., x_m\} \subseteq \text{aff}(\Delta_m) \subseteq \text{aff}(C)$.

Since $x_0, ..., x_m$ are affinely independent, $\dim\{x_0, ..., x_m\} = m = \dim(C)$.

Then $\text{aff}\{x_0, ..., x_m\} = \text{aff}(C)$ by dimension argument.

Hence, $\text{aff}\{x_0, ..., x_m\} = \text{aff}(\Delta_m) = \text{aff}(C)$.

4. (a) Suppose $f(x), f(y) \leq a$. Let $\lambda \in [0, 1]$. Then

$$f(\lambda x + (1 - \lambda) y) \leq \lambda f(x) + (1 - \lambda) f(y) \leq \lambda a + (1 - \lambda) a = a.$$ 

Hence $\lambda x + (1 - \lambda) y \in \{x \in \mathbb{R}^n | f(x) \leq a\}$ and the level set is convex.

(b) No. Consider $f(x) = x^2$. $C = (1, 2)$. 