

## Math4230 Tutorial 2

1. (a) Let  $C$  be a nonempty, convex set that contains 0 and let  $0 \leq \lambda_1 \leq \lambda_2$ . Show that  $\lambda_1 C \subseteq \lambda_2 C$ .  
  
(b) Let  $C$  be a nonempty, convex set. Let  $\alpha, \beta \geq 0$ . Show that  $\alpha C + \beta C \subseteq (\alpha + \beta)C$ .
2. Let  $\{x_1, \dots, x_m\}$  be affinely independent and suppose  $x \notin \text{aff}(\{x_1, \dots, x_m\})$ . Show that  $x_1, \dots, x_m, x$  are affinely independent.
3. Suppose  $C$  is convex with  $\dim(C) = m$ ,  $m \geq 1$ . Let  $\{x_0, x_1, \dots, x_m\} \subset C$  be an affinely independent set. Let  $\Delta_m := \text{conv}(\{x_0, x_1, \dots, x_m\})$ . Show that  $\text{aff}(C) = \text{aff}(\Delta_m) = \text{aff}(\{x_0, x_1, \dots, x_m\})$ .
4. Let  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  be a convex function.
  - (a) Show that for every  $a \in \mathbb{R}$  the *level set*  $\{x \in \mathbb{R}^n \mid f(x) \leq a\}$  is convex.
  - (b) Let  $C \subseteq \mathbb{R}$  be a convex set. Is it true in general that the inverse image  $f^{-1}(C)$  is a convex set?