

THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH 3030 Abstract Algebra 2019-20
Homework 7
Due Date: 31st October 2019

Compulsory part

1. Let G be a finite group and let primes p and $q \neq p$ divide $|G|$. Prove that if G has precisely one proper Sylow p -group, it is a normal subgroup, so G is not simple.
2. Let G be a finite group and let p be a prime dividing $|G|$. Let P be a Sylow p -group of G . Show that $N[N[P]] = N[P]$.
3. Show that every group of order $(35)^3$ has a normal subgroup of order 125.
4. Show that there are no simple groups of order $p^r m$, where p is a prime, r is a positive integer greater than 1, and $m < p$.
5. Prove that every group of order $(5)(7)(47)$ is abelian and cyclic.
6. Show that every group of order 30 contains a subgroup of 15.