MATH2010E: Advanced Calculus I
Examples and graphs

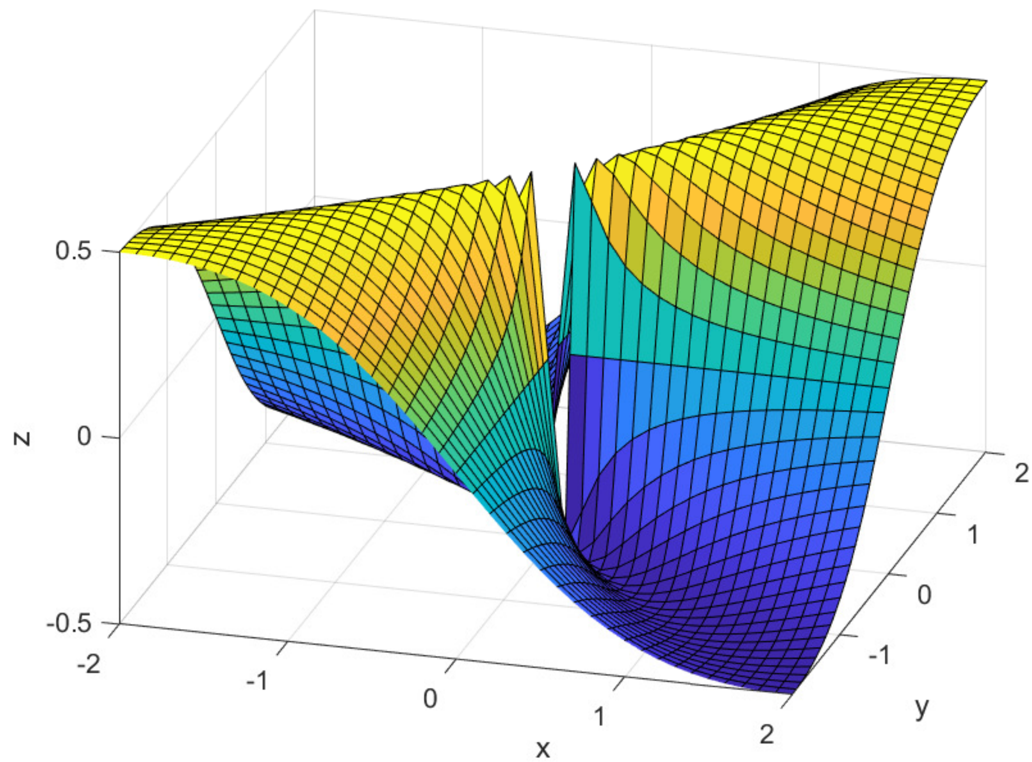

Figure 1: Caption

This is the graph $z = f(x, y)$, with $f(x, y)$ defined by

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

This function is an example that when you approach the origin along different straight lines, you would get different limits.

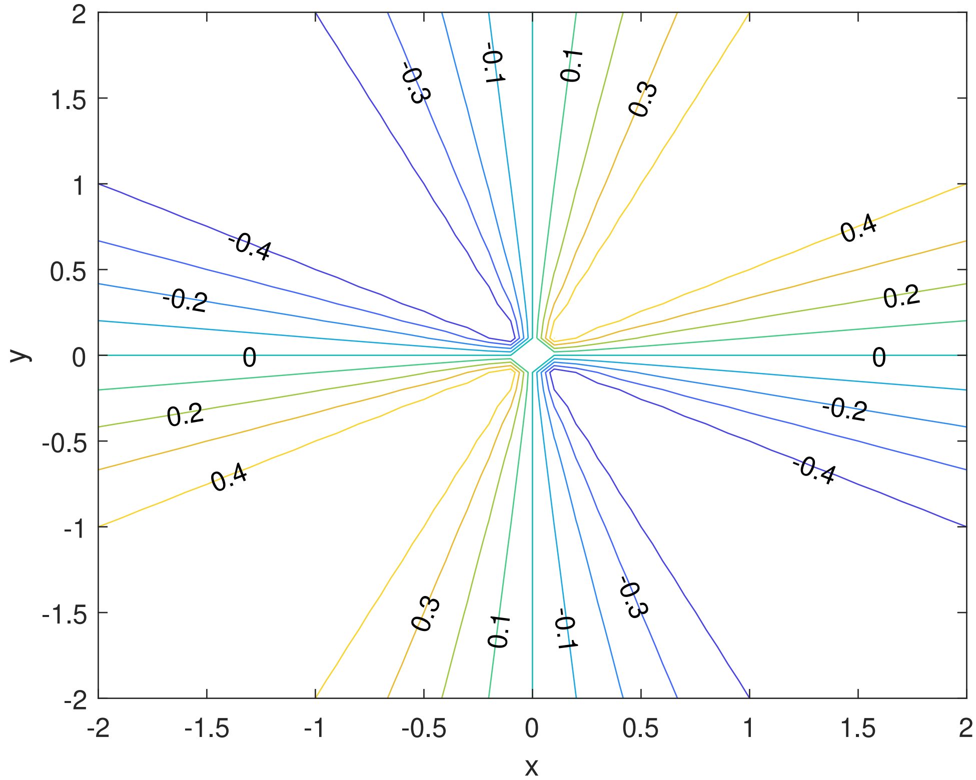

Figure 2: Caption

The curve labeled with c is the curve $f(x, y) = c$, where c is between -0.4 and 0.4. The function f is given as above. These curves are called level curves. From this graph, you may see that you can have (x_1, y_1), (x_2, y_2) near the origin with $f(x_1, y_1) = 0.4$ and $f(x_2, y_2) = -0.4$. This suggests that the function f is discontinuous at the origin.

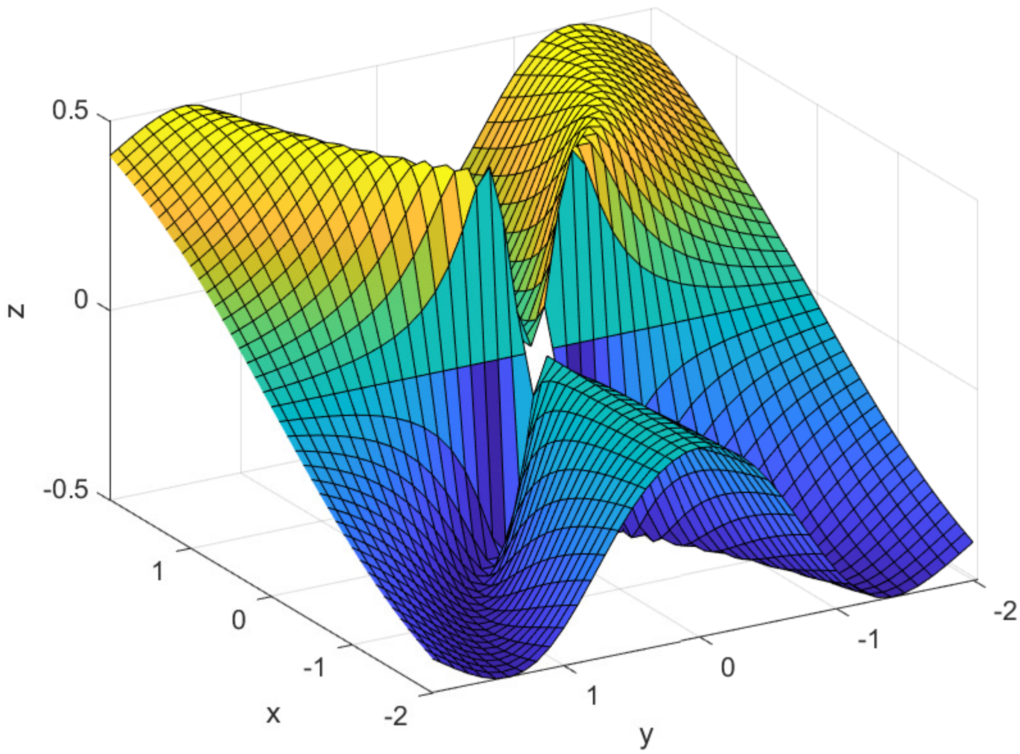

Figure 3: Caption

This is the graph $z = g(x, y)$, with $g(x, y)$ defined by

$$g(x,y) = \begin{cases} \frac{xy^2}{x^2+y^4} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

This function is an example that when you approach the origin along different straight lines, you would get the same limit 0, but the function itself is discontinuous at the origin.

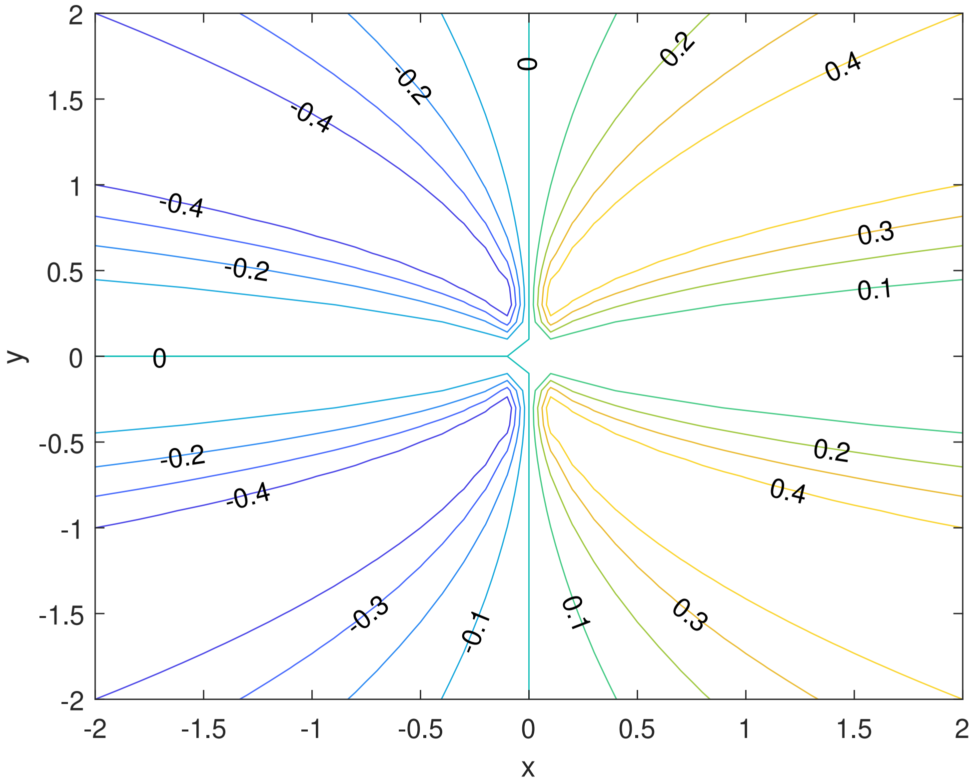

Figure 4: Caption

The curve labeled with c is the curve $g(x, y) = c$, where c is between -0.4 and 0.4. The function g is the one in the previous figure. The function g is discontinuous at the origin.

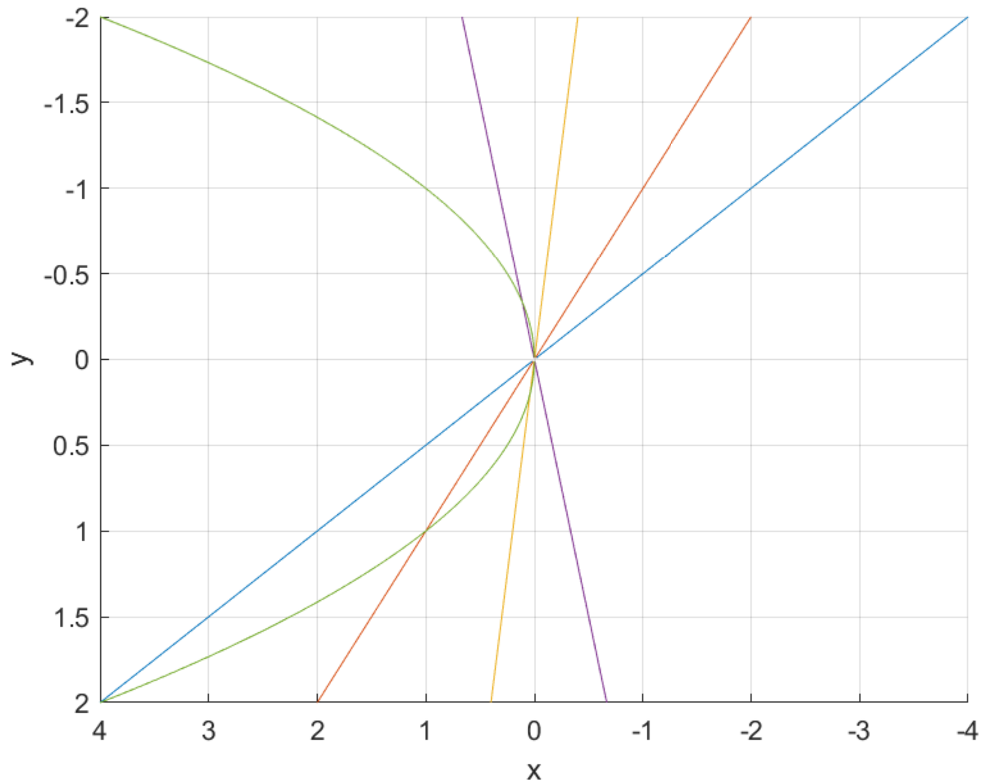

Figure 5: Caption

Please compare this figure with the one on the next page. They are the same graphs. This one is viewing from the positive z-axis. The green curve is $x = y^2$. All curves are lying on the surface $z = g(x, y)$. We have chosen different curves to approach $(0, 0)$ in the xy-plane, and see what is happening on the surface.

These two figures imitate the result that

1. If you approach the origin along a straight line, then $g(x, y)$ will get closer and closer to 0.
2. If you approach the origin along the curve $x = y^2$, $g(x, y)$ will take constant value 0.5 along the whole curve.

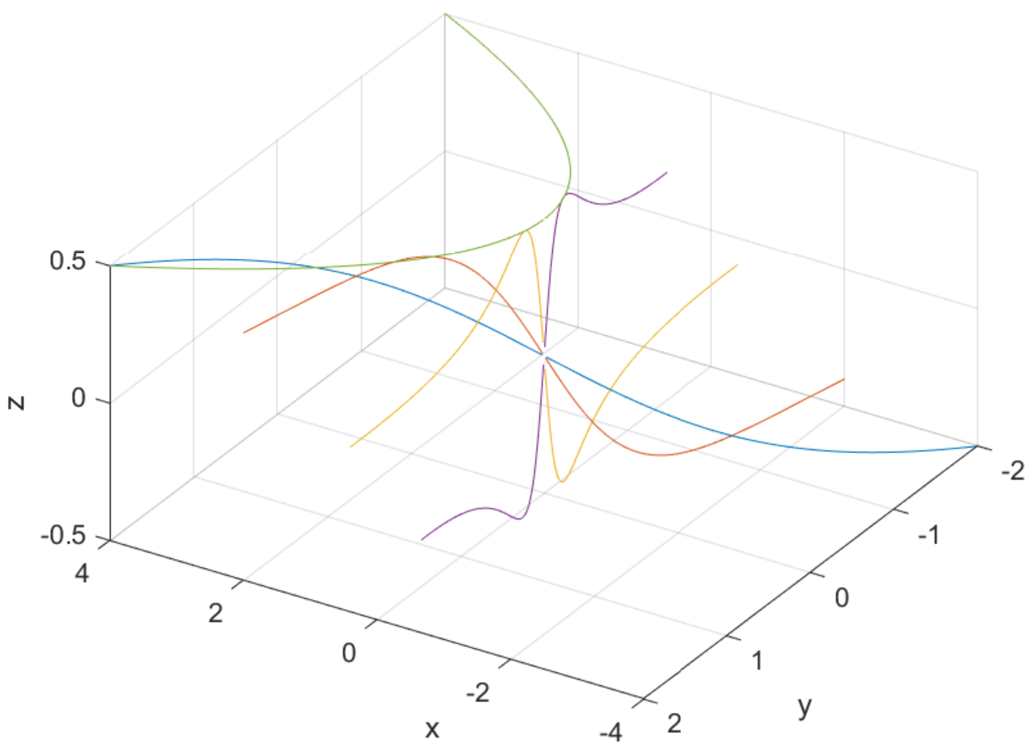

Figure 6: Caption