

Tutorial 1

This tutorial focuses mainly on Cauchy-Schwarz inequality.

- ① (a) Show the (\Leftarrow) part of Cauchy-Schwarz inequality \Leftrightarrow triangle inequality.
- (b) Is it true that "Cauchy-Schwarz equality \Leftrightarrow triangle equality"?

Ans:

Ans: Suppose it is true that

$$\forall \vec{a}, \vec{b} \in \mathbb{R}^n, \|\vec{a} + \vec{b}\| \leq \|\vec{a}\| + \|\vec{b}\|$$

Let $\vec{x}, \vec{y} \in \mathbb{R}^n$.

$$\|\vec{x} + \vec{y}\| \leq \|\vec{x}\| + \|\vec{y}\|$$

$$\|\vec{x} + \vec{y}\|^2 \leq (\|\vec{x}\| + \|\vec{y}\|)^2$$

$$(\vec{x} + \vec{y}) \cdot (\vec{x} + \vec{y}) \leq \|\vec{x}\|^2 + 2\|\vec{x}\| \cdot \|\vec{y}\| + \|\vec{y}\|^2$$

$$\vec{x} \cdot \vec{x} + 2\vec{x} \cdot \vec{y} + \vec{y} \cdot \vec{y} \leq \|\vec{x}\|^2 + 2\|\vec{x}\| \cdot \|\vec{y}\| + \|\vec{y}\|^2$$

$$\vec{x} \cdot \vec{y} \leq \|\vec{x}\| \cdot \|\vec{y}\|$$

