HOMEWORK IV (DEADLINE : 14ST JUNE, 2019)

I. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -3 & 2 & 4 \end{pmatrix}$.

(a) Find a Jordan decomposition of A.
(b) Find the general solution to the O.D.E. $x'(t) = Ax(t)$.

II. Let $A = \begin{pmatrix} 5 & -3 & -2 \\ 8 & -5 & -4 \\ -4 & 3 & 3 \end{pmatrix}$.

(a) Find a Jordan decomposition of A.
(b) Find the general solution to the O.D.E. $x'(t) = Ax(t)$.

III. (a) Let ω be a real number. Consider the initial value problem $y'' + \omega^2 y' = 0$, $y(0) = y_0$, $y_1(0) = y_1$. Let $x_1 = y$, $x_2 = y'$. For $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $x' = Ax$. Find the matrix A and solve the initial value problem by finding e^{tA}.

(b) Find the general solution to the following O.D.E.

$$x' = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix} x + \begin{pmatrix} e^t \\ t \end{pmatrix}.$$

V. Let $x_1 = y$, $x_2 = y'$, then the second order equation

$$y^{(2)} + p(t)y' + q(t)y = 0 \tag{1}$$

corresponds to the system

$$\begin{aligned} x_1' &= x_2, \\ x_2' &= -q(t)x_1 - p(t)x_2. \end{aligned} \tag{2}$$

Show that if $\{y_1, y_2\}$ and $\{\phi_1, \phi_2\}$ are fundamental sets of equation (1) and (2), respectively, then

$$W(y_1, y_2)(t) = c\hat{W}(\phi_1, \phi_2)(t),$$

where c is a non-zero constant and W and $\hat{W}$ denote the Wronskian functions given by $W(y_1, y_2) = \det \begin{pmatrix} y_1 & y_2 \\ y_1' & y_2' \end{pmatrix}$ and $\hat{W}(\phi_1, \phi_2) = \det(\phi_1, \phi_2)$.