HOMEWORK III (DEADLINE : 7ST JUNE, 2019)

I. Find the solution to the following O.D.E. with given initial value.
   (i) $y^{(3)} + y' = \sec t,\ \ y(0) = 2,\ y'(0) = 1,\ y^{(2)}(0) = -2.$
   (ii) $y^{(4)} + 2y^{(2)} + y = \cos t$, $y(0) = 2$, $y'(0) = 0$, $y^{(2)}(0) = -1$, $y^{(3)}(0) = 1$.
   (iii) $y^{(3)} - y^{(2)} + y' - y = \sec t$, $y(0) = 2$, $y'(0) = -1$, $y^{(2)}(0) = 1$.
   (iv) $y^{(4)} - y^{(3)} - y^{(2)} + y' = t^2 + 8 + t \sin t$, $y(0) = 1$, $y'(0) = 1$, $y^{(2)}(0) = 1$, $y^{(3)}(0) = 1$.

II. Find a formula involving integrals for a particular solution of the O.D.E.

$$y^{(3)} - 3y^{(2)} + 3y' - y = g(t).$$

If $g(t) = t^{-2}e^t$, determine $Y(t)$.

III. If $\varphi_1(t)$ is a solution to the O.D.E.

$$y^{(3)}(t) + p_2(t)y^{(2)}(t) + p_1(t)y'(t) + p_0(t)y(t) = 0.$$

   (i) Assume that $y(t)$ is a solution of the O.D.E. above. Use the substitution $y(t) = \varphi_1(t)v(t)$ to derive a second order O.D.E. for v'.
   (ii) Use (i) to find the general solution for the O.D.E.

$$(2 - t)y^{(3)}(t) + (2t - 3)y^{(2)}(t) - ty^{(1)}(t) + y(t) = 0$$

   for $t < 2$ provided that $\varphi_1(t) = e^t$ is a solution.

IV. Let $\{\varphi_1, \dots, \varphi_n\}$ be a collection of solutions of the O.D.E.

$$y^{(n)}(t) + p_{n-1}(t)y^{(n-1)}(t) + \cdots + p_1(t)y'(t) + p_0(t)y(t) = 0,$$

where $p_j(t)$ is continuous on some interval I. Assume that $\{\varphi_1, \dots, \varphi_n\}$ is linearly independent on I. Snow that

$$W(\varphi_1, \dots, \varphi_n)(t) \neq 0, \quad \text{for every } t \in I.$$

Hint: Using the fact that there is a fundamental set of the O.D.E. above.

V. Let r_1, r_2 be two distinct real numbers, let a, b be real numbers with $b \neq 0$. Show that the collection of functions

$$\left\{e^{r_1 t}, te^{r_2}, t^2 e^{r_2}, e^{at}\cos bt, e^{at}\sin bt\right\}$$

are linearly independent.