HOMEWORK II (DEADLINE : 31ST MAY, 2019)

I. Find the general solution to the following O.D.E.

(i) $y'' + 8y' - 9y = 0$.

(ii) $y'' + 4y' + 4y = 0$.

(iii) $y'' + 5y' + 8y = 0$.

II. (i) If the Wronskian of any two solutions of $y''+p(t)y'+q(t)y = 0$ is constant, what does this imply about the coefficients $p(t)$ and $q(t)$?

(ii) If f, g and h are differential functions, show that $W(fg, fh) = f^2W(g, h)$.

III. Find the Wroskian of the two solutions of the following O.D.E. without solving the O.D.E.

(i) $x^2y'' + xy' + (x^2 - v^2)y = 0$, where v is a constant (Bessel's equation).

(ii) $(1 - x^2)y'' - 2xy' + \alpha(\alpha + 1)y = 0$, where α is a constant (Legendre's equation).

IV. In the following O.D.E., find a second independent solution of the given O.D.E.

(i) $t^2y'' + 3ty' + y = 0$, $t > 0$; $y_1(t) = t^{-1}$.

(ii) $(x - 1)y'' - xy' + y = 0$, $x > 1$; $y_1(x) = e^x$.

V. Find the general solution to the following O.D.E.

(i) $y'' + 3y' + 2y = e^t(t^2 + 1)\sin 2t + 3e^{-t}\cos t + 6e^t$.

(ii) $y'' + 2y' + 5y = 3te^{-t}\cos 2t - 2te^{-2t}\cos t$.