HOMEWORK I (DEADLINE : 24ST MAY, 2019)

I. Solve the following initial value problems:
   (i) $t^4y' + 5t^3y = e^{-t}, y(-1) = 0$, for $t < 0$.
   (ii) $(sint)y' + (cost)y = 2e^t, y(1) = a, 0 < t < \pi$.

II. Solve the initial value problem

$$y' = \frac{1+3x^2}{3y^2-12y}, \quad y(0) = 1$$

and determine the interval in which the solution is valid.

III. Solve the following O.D.E.
   (i)

$$\frac{dy}{dx} = \frac{ay+b}{cy+d},$$

   where a, b, c, d are constants.
   (ii) $y + (2t - 3ye^y)y' = 0$, $y(1) = 0$.

IV. Determine whether each of the following equations is exact. If it is exact, find the solution.
   (i) $(e^x siny - 3ysinx) + (e^x cosy + 3cosx)y' = 0$.
   (ii) $(x\log y + xy) + (y\log x + xy)y' = 0$, $x > 0$, $y > 0$.
   (iii) $\frac{x}{(x^2+y^2)^{\frac{3}{2}}} + \frac{y}{(x^2+y^2)^{\frac{3}{2}}}\frac{dy}{dx} = 0$.

V. Show that if $\frac{(N_x - M_y)}{(xM - yN)} = R$, where R depends on the quantity xy only, then the differential equation

$$M + Ny' = 0$$

has an integrating factor of the form $\mu(xy)$. Fin a general formula for this integrating factor.