HOMEWORK II (DEADLINE : 12ND OCTOBER, 2018)

ORDINARY DIFFERENTIAL EQUATIONS

Answer all questions:

(1) (4 points) **Find** the general solution to the following differential equations:

(a) $y'' + 8y' - 9y = 0$;

(b) $9y'' + 16y = 0$;

(c) $y'' + 4y' + 4y = 0$;

(d) $y'' + 2y' + y = 4e^{-t}$;

(e) $y'' - 2y' - 3y = 3te^{2t}$;

(f) $2y'' + 3y' + y = t^2 + 3\cos(t)$;

(h) $y'' + 2y' + 5y = 4e^{-t}\cos(2t)$.

(i) $t^2y'' + 7ty' + 5y = 3t$, for $t > 0$ provided that $y_1(t) = t^{-1}$ is a solution to the homogeneous equation $t^2y'' + 7ty' + 5y = 0$.

(2) (1 point) **Determine** a suitable form of $Y(t)$ for using the method of undetermined coefficients to the following equations (you don't have to solve the equation explicitly, but simply guess the form of $Y(t)$ to be used):

(a) $y'' - 4y' + 4y = 4t^2 + 4te^{2t} + t\sin(2t)$;

(b) $y'' + 3y' + 2y = e^t(t^2 + 1)\sin(2t) + 3e^{-t}\cos(t) + 6e^t$.

(3) (1 point) If $y(t)$ is a solution of the differential equation $y'' + p(t)y' + q(t)y = r(t)$, where $r(t)$ is not always zero, **show** that $cy(t)$ is not a solution for the equation for $c \neq 1$.

(4) (1 point) **Can** $y(t) = \sin(t^2)$ be a solution on an interval $I = (-\delta, \delta)$ to the equation $y'' + p(t)y' + q(t)y = 0$ with continuous coefficients $p(t)$ and $q(t)$? **Explain** your answer.

(5) (1 point) Assume that y_1 and y_2 is a fundamental set of solutions to $y'' + p(t)y' + q(t)y = 0$, and we let $y_3 = a_1y_1 + a_2y_2$ and $y_4 = b_1y_1 + b_2y_2$, where a_1, a_2, b_1, b_2 are constants. **Show** that

$$W(y_3, y_4) = (a_1b_2 - a_2b_1)W(y_1, y_2),$$

and **determine** the condition that y_3, y_4 is again a fundamental set of solutions.

(6) (1 point) Given f, g, h be continuously differentiable function on an interval I, **show** that

$$W(fg, fh) = f^2W(g, h).$$

(7) (2 points) Consider an equation of the form

$$t^2\frac{d^2y}{dt^2} + \alpha t\frac{dy}{dt} + \beta y = 0, \tag{0.1}$$

for $t > 0$, where α, β are real constants.

(a) Using the subsitution $x = \log(t)$, **compute** the terms $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of $\frac{dy}{dt}$ and $\frac{d^2y}{dt^2}$.

(b) Use the above subsitution to **transform** the equation into

$$\frac{d^2y}{dx^2} + (\alpha - 1)\frac{dy}{dx} + \beta y = 0, \tag{0.2}$$

and **conclude** that $y_1(\log(t)), y_2(\log(t))$ is a fundamental set of solutions to equation (0.1) if $y_1(x), y_2(x)$ is a fundamental set of solutions to equation (0.2).

(8) (2 points) Consider the equation

$$ay'' + by' + cy = 0.$$

(a) If all a, b, c are positive constants, **show** that all the solutions of the equation approach 0 as $t \to \infty$.

(b) If $a > 0$, $c > 0$ but $b = 0$, **show** that all the solutions are bounded as $t \to \infty$.

(9) (2 points) Consider the differential equation

$$y'' + y = r(t).$$

(a) **Deduce** that

$$Y(t) := \int_{t_0}^{t} \sin(t - s)r(s)ds$$

is a solution to the initial value problem $y(t_0) = 0$, $y'(t_0) = 0$ using integral formula obtain from method of variation of parameter for non-homogeneous equation.

(b) **Find** the solution of the initial value problem $y(t_0) = 1$, and $y'(t_0) = 2$ in terms of the particular solution $Y(t)$.

End of Homework 1